Thermodynamics Basics:

Phases of Water, Laws and Processes

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Preface

As the adage goes, "a picture is worth a thousand words;" this book maximizes the utilization of diagram, graphs and flow charts to facilitate quick and effective comprehension of the concepts of thermodynamics by the reader.

This book is designed to serve as a tool for building basic engineering skills in the filed of thermodynamics.

If your objective as a reader is limited to the acquisition of basic knowledge in thermodynamics, then the material in this book should suffice. If, however, the reader wishes to progress their knowledge and skills in thermodynamics to intermediate or advance level, this book could serve as a useful stepping stone.

In this book, the study of thermodynamics concepts, principles and analysis techniques is made relatively easy for the reader by inclusion of most of the reference data, in form of excerpts, within the discussion of each case study, exercise and self-assessment problem solutions. This is in an effort to facilitate quick study and comprehension of the material without repetitive search for reference data in other parts of the book.

Certain thermodynamic concepts and terms are explained more than once as these concepts appear in different segments of this text; often with a slightly different perspective. This approach is a deliberate attempt to make the study of some of the more abstract thermodynamics topics more fluid; allowing the reader continuity, and precluding the need for pausing and referring to segments where those specific topics were first introduced.

Due to the level of explanation and detail included for most thermodynamics concepts, principles, computational techniques and analyses methods, this book is a tool for those energy engineers, engineers and non-engineers, who are not current on the subject of thermodynamics.

The solutions for end of the segment self-assessment problems are explained in just as much detail as the case studies and sample problem in the pertaining segments. This approach has been adopted so that this book can serve as a

thermodynamics skill building resource for not just energy engineers but engineers of all disciplines. Since all segments and topics begin with the introduction of important fundamental concepts and principles, this book can serve as a "brush-up" or review tool for even mechanical engineers whose current area of engineering specialty does not afford them the opportunity to keep their thermodynamics knowledge current.

In an effort to clarify some of the thermodynamic concepts effectively for energy engineers whose engineering education focus does not include thermodynamics, analogies are drawn from non-mechanical engineering realms, on certain complex topics, to facilitate comprehension of the relatively abstract thermodynamic concepts and principles.

Each segment in this book concludes with a list of questions or problems, for self-assessment, skill building and knowledge affirmation purposes. The reader is encouraged to attempt these problems and questions. The answers and solutions, for the questions and problems, are included under Appendix A of this text.

For reference and computational purposes, steam tables and Mollier (Enthalpy-Entropy) diagrams are included in Appendix B.

Most engineers understand the role units play in definition and verification of the engineering concepts, principles, equations and analytical techniques. Therefore, most thermodynamic concepts, principles and computational procedures covered in this book are punctuated with proper units. In addition, for the reader's convenience, units for commonly used thermodynamic entities, and some conversion factors are listed under Appendix C.

Most thermodynamic concepts, principles, tables, graphs, and computational procedures covered in this book are premised on US/Imperial Units as well as SI/Metric Units. Certain numerical examples, case studies or self-assessment problems in this book are premised on *either* the SI unit realm *or* the US unit system. When the problems or numerical analysis are based on only one of the two unit systems, the given data and the final results can be transformed into the desired unit system through the use of unit conversion factors in Appendix C.

Some of the Greek symbols, used in the realm of thermodynamics, are listed in Appendix D, for reference.

What readers can gain from this book:

- Better understanding of thermodynamics terms, concepts, principles, laws, analysis methods, solution strategies and computational techniques.

- Greater confidence in interactions with thermodynamics design engineers and thermodynamics experts.

- Skills and preparation necessary for succeeding in thermodynamics portion of various certification and licensure exams, i.e. CEM, FE, PE, and many other trade certification tests.

- A better understanding of the thermodynamics component of heat related energy projects.

- A compact and simplified thermodynamics desk reference.

Table of Contents

Segment 1

Phases of Water and Associated Thermodynamics

Topics:
- Thermodynamic phases
- Critical properties of substances, with emphasis on water.

Phases of Substance:

Thermodynamic properties and phases of a substance are defined and determined by two important properties; namely, temperature and pressure.

Three most common phases of a substance are as follows:

1) Solid

2) Liquid

3) Gaseous

These three phases, subcategories within these phases, and other pertinent thermodynamic terms are defined below.

Solid:

The shape and volume of a substance, in solid phase, is non-volatile. A substance in its solid phase does not adapt itself to the shape or volume of its container. The temperature – under atmospheric pressure conditions - at which a substance attains the solid phase, is called the ***freezing point***.

Liquid:

The shape of a substance, in liquid phase, is volatile. A substance in its liquid phase adapts itself to the shape or of its container. The temperature – under atmospheric pressure conditions - at which a substance attains the liquid phase is called the ***melting point***.

Gas:

The shape and volume of a substance, in gaseous phase, is volatile. Gas is a state of matter consisting of a collection of molecular or atomic particles that lacks definite shape or volume. The temperature – under atmospheric pressure conditions - at which a substance attains the gaseous phase, is called the *boiling point*.

Sensible Heat:

Sensible heat is the heat required, or absorbed, in raising the temperature of a substance, without a change in phase. **Example:** Heat required to raise the temperature of water from 60°F to 80°F, at sea level, or a pressure of 1 atm. Calculation of sensible heat is demonstrated in other segments.

Latent Heat:

Latent heat is the heat required or absorbed in changing the phase of a substance. Latent heats for fusion, sublimation and vaporization of water are listed (in SI/Metric Units and US/Imperial Units) in Table 1.1. Consider the following examples as illustration of how to use this table:

Example A: Latent Heat of Fusion, h_{SL}, which is the heat required to fully melt ice to liquid water, is **334 kJ/kg**.

Example B: Latent Heat of Sublimation, h_{SV}, which is the heat required to fully evaporate ice to saturated vapor phase, is **2838 kJ/kg**.

Example C: Latent Heat of Vaporization, h_{fg}, which is the heat required to fully evaporate water, at sea level - or a pressure of 1 atm - and 100 °C, is **2260 kJ/kg**.

	Latent Heat Fusion kJ/kg	Latent Heat Sublimation kJ/kg	Latent Heat Vaporization kJ/kg
kJ/kg	334	2838	2260 at 100ºC
BTU/lbm	143.4	1220	970.3
kcal/kg	79.7	677.8	539.1

Table 1.1, Latent Heats for Water Phase Transformation, SI Units

Saturation Temperature:

Saturation temperature, at a given pressure, is the temperature below which a gas or vapor would condense to liquid phase. For example, the saturation temperature at standard atmospheric pressure of 101 kPa (0.1014MPa) is 100°C. See the circled pressure and temperature values in Table 1.2. Now, if the pressure is reduced to 12.4 kPa (0.0124 MPa), the saturation temperature would drop to 50°C. In other words, if the pressure is reduced to 12.4 kPa (0.0124 MPa), the water's boiling point would be reduced to 50°C.

Saturation Pressure:

Saturation pressure, at a given temperature, is the pressure above which a gas or vapor would condense to liquid phase. For example, the saturation pressure at a temperature of 100°C is 101 kPa (0.1014MPa). See the circled pressure and temperature values in Table 1.1. Now, if the pressure is increased to (1.5547 MPa), the saturation temperature rises to 200 °C. In other words, if the pressure is increased to 1,554.7 kPa, the water's boiling point would double, from 100°C to 200 °C. This also means that if the initial temperature and pressure conditions are 100°C and 101 kPa, and the pressure is escalated by almost 15 fold, up to 1,5547 kPa, the water would no longer be in saturated water phase; it will instead fall back into subcooled liquid phase. See the discussion on subcooled liquids in the next section.

Same example in the US unit realm would be that of saturation pressure at a given temperature of say 209 °F. The saturation pressure at the given

temperature of 209 °F, as circled in Table 1.2, would be 14 psia. If, however, the pressure is raised, for instance, to 100 psia, the saturation temperature would rise to 328 °F. In other words, if the initial conditions are changed such that while the temperature remains the same, i.e. 209 °F, the pressure is increased from 14 psia to 100 psia, the water would fall into the subcooled liquid state.

Subcooled Liquid:

When the temperature of a liquid is less than its boiling point, at a given pressure, it is referred to be in a subcooled state or phase. **Example:** Water at room temperature (77 °F, 25°C), at sea level (1-Atm or 1-Bar), is considered to be subcooled, in that, addition of a small amount of heat will not cause the water to boil.

- If, at a certain pressure, temperature is a determining variable, a substance is said to be in a subcooled liquid phase when its temperature is *below* the saturated temperature value corresponding to its pressure.

- Conversely, if at a certain temperature, pressure is a determining variable, a substance is said to be in a subcooled liquid phase when its pressure is *greater* than the saturated pressure value corresponding to that temperature.

Saturated Liquid: When the temperature of a liquid is almost at its boiling point, such that addition of a small amount of heat energy would cause the liquid to boil, it is said to be saturated. In other words, it is saturated with heat and cannot accept additional heat without evaporating into vapor phase. **Example:** Water at 212 °F, or 100°C, at sea level.

Saturated Vapor:
Vapor that has cooled off to the extent that it is almost at the boiling point, or saturation point, and at the verge of condensing, is called a saturated vapor.

Liquid-Vapor Phase:
A substance is said to be in a liquid-vapor phase when its temperature is equal to the saturated temperature value corresponding to its pressure. When water

is in the liquid-vapor phase, in most cases, a portion of the total volume of water has evaporated; the remaining portion is in saturated water state.

Superheated Vapor:
Superheated vapor is vapor that has absorbed heat beyond its boiling point. Loss of a small amount of heat would not cause superheated vapor to condense.

- If, at a certain pressure, temperature is the determining variable, then a vapor is said to be in a superheated vapor state when its temperature *exceeds* the saturation temperature corresponding to its pressure.

- Conversely, if at a certain temperature, pressure is the determining variable, a vapor is said to be in a superheated vapor state when its pressure is *less* than the saturation pressure corresponding to that temperature.

Example Problem 1:
Answer the following questions for water at a temperature of 153 °F and pressure of 4 psia:

a) Heat content for saturated water.

b) Specific heat (BTU/lbm) required to evaporate the water.

c) If the water were evaporated, what would the saturated vapor heat content be?

d) What state or phase would the water be in at the stated temperature and pressure?

e) What would the entropy of the water be while it is in a saturated liquid phase?

f) What would the specific volume of the water be while it is in a saturated vapor phase?

Solution/Answers:

a) At 153°F and pressure of 4 psia, the water is in a saturated liquid form. According to saturated steam table excerpt in Table 1.3, the saturated water enthalpy is 120.8 BTU/lbm. This value is listed under column labeled h_L, in Table 1.3, in the row representing temperature of 153°F and pressure of 4 psia.

Answer: The enthalpy or heat content for saturated water, at the given temperature and pressure, is **120.8 BTU/lbm**.

b) The specific heat, in BTU/lbm, required to evaporate the water from saturated liquid phase to saturated vapor phase, is represented by the term **h_{fg}**. The value of h_{fg}, for saturated water at 153°F and a pressure of 4 psia, as read from Table 1.3, is 1006 BTU/lbm. See circled values in Table 1.3.

Answer: **$h_{fg \text{ at} 153°F \text{ and } 4 \text{ psia}}$ = 1006 BTU/lbm**

c) Saturated vapor heat content, if the water were evaporated, would be the value for **$h_{v \text{ at} 153°F \text{ and } 4 \text{ psia}}$,** and from Table 1.3 this value is 1127 BTU/lbm.

Answer: **$h_{v \text{ at} 153°F \text{ and } 4 \text{ psia}}$ = 1127 BTU/lbm**

d) The water would be in **saturated liquid phase** at the stated temperature and pressure. All stated saturation temperatures and pressures, in the saturated steam tables, represent the current state of water in saturated liquid phase.

Answer: **Saturated liquid phase**

e) The entropy of water at 153°F and a pressure of 4 psia, in saturated liquid phase, as read from Table 1.3, would be s_L= 0.22 BTU/(lbm.°R). Note that s_L value is retrieved form the table and not the s_V value. This is because the problem statement specifies the *liquid phase.*

Answer: s_L= 0.22 BTU/(lbm.°R)

f) The specific volume of water at 153°F and a pressure of 4 psia, in saturated vapor phase, as read from Table 1.3, would be v_V = 90 **ft³/lbm**. Note that v_V value is retrieved form the table and not the v_L value. This is because the problem statement specifies the *vapor phase.*

Answer: $\nu_V = 90$ **ft³/lbm**

Temp. °C	Abs. Press. MPa	Specific Volume m³/kg		Enthalpy kJ/kg			Entropy kJ/kg.°K		Temp. °C
		Sat. Liquid ν_L	Sat. Vapor ν_V	Sat. Liquid h_L	Evap. h_{fg}	Sat. Vapor h_V	Sat. Liquid s_L	Sat. Vapor s_V	
20	0.0023	0.001	57.76	83.9	2454	2537	0.30	8.67	20
50	0.0124	0.001	12.02	209	2382	2591	0.70	8.07	50
100	0.1014	0.001	1.672	419	2257.	2675	1.31	7.35	100
200	1.5547	0.001	0.127	852	1940	2792	2.33	6.43	200

Title row: **Properties of Saturated Steam By Temperature** — **Metric/SI Units**

Table 1.2, Properties of Saturated Steam by Temperature, SI Units

Same example in the US unit realm would be that of saturation temperature at standard atmospheric pressure of 14 psia (1 Bar). The saturation temperature at standard atmospheric pressure of 14 psia (1 Bar), as circled in Table 1.2, would be 209°F, which is approximately the same as the commonly known boiling point of water at 212°F. Converse to the last example, if the pressure is *raised* to 100 psia, the saturation temperature would escalate up to 328°F. In other words, as the pressure is increased to 100 psia, the water would boil at a much higher temperature of 328°F. See the circled temperature and pressure values in the US unit version of the saturated steam table excerpt in Table 1.3. Note that increasing the boiling point of the water by raising the pressure on the surface of the water is the same principle that is employed in pressure cookers. By raising the pressure and the temperature, water's enthalpy or heat content is raised; thus accelerating the cooking or decomposition of the contents.

Abs. Press. psia	Temp in °F	Specific Volume ft³/lbm		Enthalpy BTU/lbm			Entropy BTU/(lbm.°R)		Abs. Press. psia
		Sat. Liquid v_L	Sat. Vapor v_V	Sat. Liquid h_L	Evap. h_{fg}	Sat. Vapor h_V	Sat. Liquid s_L	Sat. Vapor s_V	
1.0	102	0.0161	333.	69.73	1036	1105	0.133	1.978	1.0
4.0	153	0.0164	90.0	120.8	1006	1127	0.220	1.862	4.0
10.0	193	0.0166	38.42	161.2	982	1143	0.2836	1.788	10.0
14.0	209	0.0167	28.0	177.6	972	1149	0.308	1.761	14.0
100	328	0.0177	4.4	298.5	889	1188	0.474	1.603	100

**Table 1.3, Properties of Saturated Steam by Pressure, US Units**

Phase Transformation of Water at Constant Pressure

The process of phase transformation of water, under constant pressure, is illustrated in **Figure 1.1**.

In Segment A, to the extreme left in **Figure 1.1**, water is depicted in subcooled liquid phase, at a certain temperature T°C, well below the saturation temperature at the given pressure. As heat is added to the water, as shown in Segment B, the temperature of the water rises to T°C + ΔT°C, thus causing the water to expand and achieve the *saturated liquid state*. This heat, that simply increases the temperature of the water without causing evaporation, is *sensible heat*.

As more heat is added into the system, some of the saturated water transforms into saturated vapor; thus resulting in a mixture of saturated water and saturated vapor phases, as shown in Segment C. The heat added to transform the saturated water into saturated vapor is *latent heat*. During this phase transformation, the water temperature stays constant, at $T_{Saturation}$. Further addition of heat leads to the transformation of all of the saturated water into saturated vapor, as shown in Segment D.

The last stage of this process is represented in Segment E of Figure 1.1. This stage shows the transformation of water from saturated vapor state to ***superheated vapor state***, through introduction of more heat. Since the water maintains the vapor state in this stage, the heat added is sensible heat.
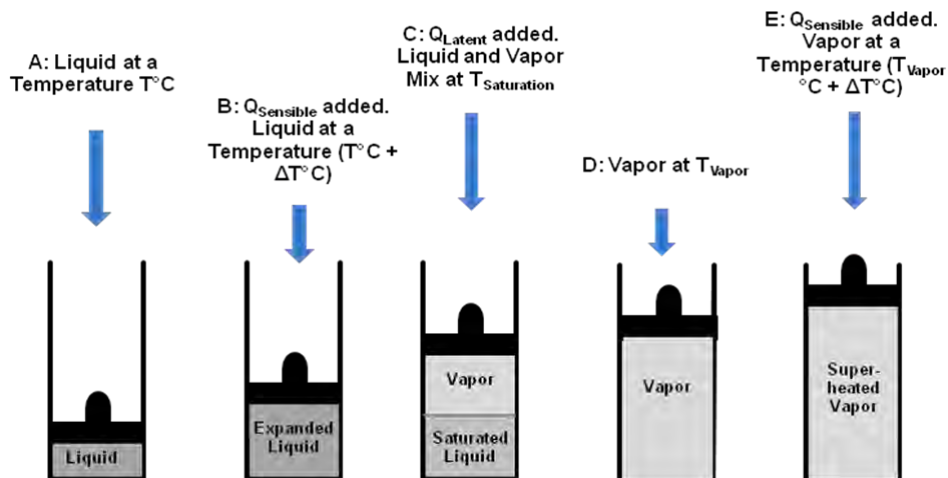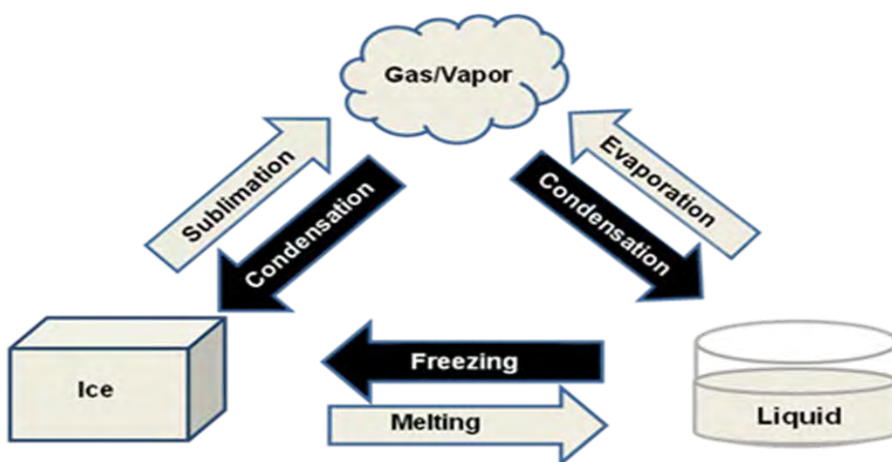

Figure 1.1 - Phase Transformation of Water at Constant Pressure

The phase transformation process described above can also be followed in graphical form as shown in Figure 1.2 below. For instance, the state of water represented by Segment A, in Figure 1.1, lies in the subcooled region, to the left of the graph in Figure 1.2. Water represented by Segment B lies directly on the saturated water line in Figure 1.2. Water represented by Ssegment C lies within the "bell curve," shown as the shaded region on the graph in Figure 1.2. Water represented by Segment D lies directly on the saturated vapor line. As more heat is added and water transitions into state represented by Segment E, it enters the superheated vapor region, shown to the right of the saturated vapor curve on Figure 1.2.
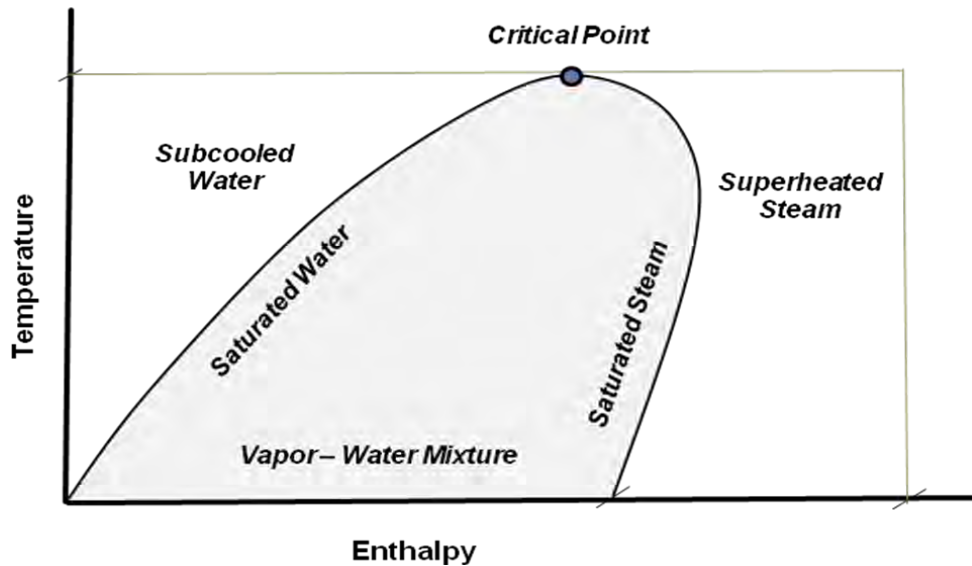
Figure 1.2 - Phase Transformation of Water at Constant Pressure, Depicted in Graphical Form

Phase and state transformation of water can also be viewed, from physical perspective, as shown in Figure 1.3. This diagram depicts the transformation of water from solid phase to vapor phase as addition of heat drives the phase transformation process, counterclockwise, from ice to vapor state. Once in vapor phase, removal of heat would drive the phase transformation cycle, clockwise, from vapor to ice phase – unless the transformation constitutes sublimation.

Phase Conversion Processes of Substances

Figure 1.3 - Phase Transformation of Water

Phase transformation from ice directly into vapor, as shown in the left half of the diagram in Figure 1.3, represents the sublimation process. Of course, sublimation can be achieved in reverse through removal of heat, resulting in direct phase transformation from vapor to solid state, or ice.

Types of Phase Transformation

Three common phase transformations, and associated latent heat values, have been discussed in earlier sections of this segment. In addition to providing tabular cross-referencing between various phases of water, Table 1.4 expands on the topic of phase transformation and lists other types of transformation and phases that don't commonly involve water. Plasma is stated as one of the phases in Table 1.4. Plasma is, essentially, ionized gas. Plasma, or ionized gas, is used in various processes, including, vaporized metal deposition on substances such as semiconductor substrates – for fabrication of integrated circuits or "chips" – and flat glass panels, for addition of energy conservation characteristics.

From	To			
	SOLID	**LIQUID**	**GAS**	**PLASMA**
SOLID	Solid-Solid Transformation	MELTING or FUSION	SUBLIMATION	N/A
LIQUID	FREEZ-ING	N/A	BOILING or EVAPORATION	N/A
GAS	DEPOSI-TION	CONDENSA-TION	N/A	IONIZA-TION
PLASMA	N/A	N/A	RECOMBINATION or DEIONIZATION	N/A

Table 1.4; Types of Phase Transformation

Ideal Gas:

Any gas that behaves in accordance with the ideal gas laws is said to be an ideal gas. Highly superheated vapor behaves like an ideal gas and is treated as such. This is demonstrated in later segments of this text.

Real Gas:

Any gas that does not behave according to the ideal gas laws is said to be a real gas. Saturated vapor is considered to be a real gas.

Critical Point:

If *temperature* and *pressure* of a liquid are increased, eventually, a state is reached where *liquid and vapor phases coexist*, and are indistinguishable. This point is referred to as a "critical point." See Point D in Figure 1.4.

Critical Properties:

Properties, such as temperature, specific volume, density and pressure of a substance at the critical point are referred to as critical properties. See Tables 1.5 and 1.6.

Triple Point:

Triple point of a substance is a state in which *solid, liquid and gaseous phases coexist.*

Substance	Critical Temperature		Critical Pressure	
	°C	°K	atm	kPa
Argon	−122.4	151	48.1	4,870
Ammonia	132.4	405.5	111.3	11,280
Bromine	310.8	584.0	102	10,300
Cesium	1,664.85	1,938.00	94	9,500
Chlorine	143.8	417.0	76.0	7,700
Fluorine	−128.85	144	51.5	5,220
Helium	−267.96	5.19	2.24	227
Hydrogen	−239.95	33.2	12.8	1,300
Krypton	−63.8	209	54.3	5,500
Neon	−228.75	44.4	27.2	2,760
Nitrogen	−146.9	126	33.5	3,390
Oxygen	−118.6	155	49.8	5,050
CO_2	31.04	304.19	72.8	7,380
H_2SO_4	654	927	45.4	4,600
Xenon	16.6	289.8	57.6	5,840
Lithium	2,950	3,220	652	66,100
Mercury	1,476.9	1,750.1	1,720	174,000
Sulfur	1,040.85	1,314.00	207	21,000
Iron	8,227	8,500	N/A	N/A
Gold	6,977	7,250	5,000	510,000
Aluminum	7,577	7,850	N/A	N/A
Water	373.946	647.096	217.7	22,060

Table 1.5 - Critical Properties of Select Substances

Substance	Triple Point Temperature, T (K)	Triple Point Pressure, (kPa)
Ammonia	195.4	6.076
Argon	83.81	68.9
Butane	134.6	7×10^{-4}
Ethane	89.89	8×10^{-4}
Ethanol	150	4.3×10^{-7}
Ethylene	104	0.12
Hydrogen	13.84	7.04
Hydrogen Chloride	158.96	13.9
Methane	90.68	11.7
Nitric oxide	109.5	21.92
Nitrogen	63.18	12.6
Nitrous Oxide	182.34	87.85
Oxygen	54.36	0.152
Water	273.16	0.6117
Zinc	692.65	0.065

Table 1.6 – Triple Point Properties of Select Substances

Comparison - Triple Point vs. Critical Point:

Both triple point and critical point have been explained earlier in sections of this segment. In this section, we will compare these two points for clarification and distinction. Figure 1.4, is used to illustrate the differences between these two points. The graph depicted in Figure 1.4 plots pressure as a function of temperature. Note that water – or any other substance in question – can exist in liquid or gaseous phases, simultaneously, to the right of the line labeled A-B. And, since the critical point "D" lies in the region to the right of line A-B, critical point can only involve two phases, namely, the gaseous phase and the liquid phase. Also, note that T_{Cr}, the temperature at the critical point – or critical temperature - is substantially higher than the, the triple point temperature. At this higher temperature T_{Cr}, solid phase cannot exist, for most practical purposes. On the other hand, at the triple point, substance exists in all

three phases: solid, liquid and gaseous – simultaneously. As shown in Figure 1.4, triple point C lies in the region where solid, liquid and gaseous phases coexist. The solid phase is to the left of the dashed line E-C, liquid phase in the B-C-D region and vapor phase lies to the right of the line formed by points A, C and D.

Another key distinction between triple point and critical point, from pressure point of view, is that critical point pressure, P_{Cr}, is significantly higher than the critical pressure, P_t.
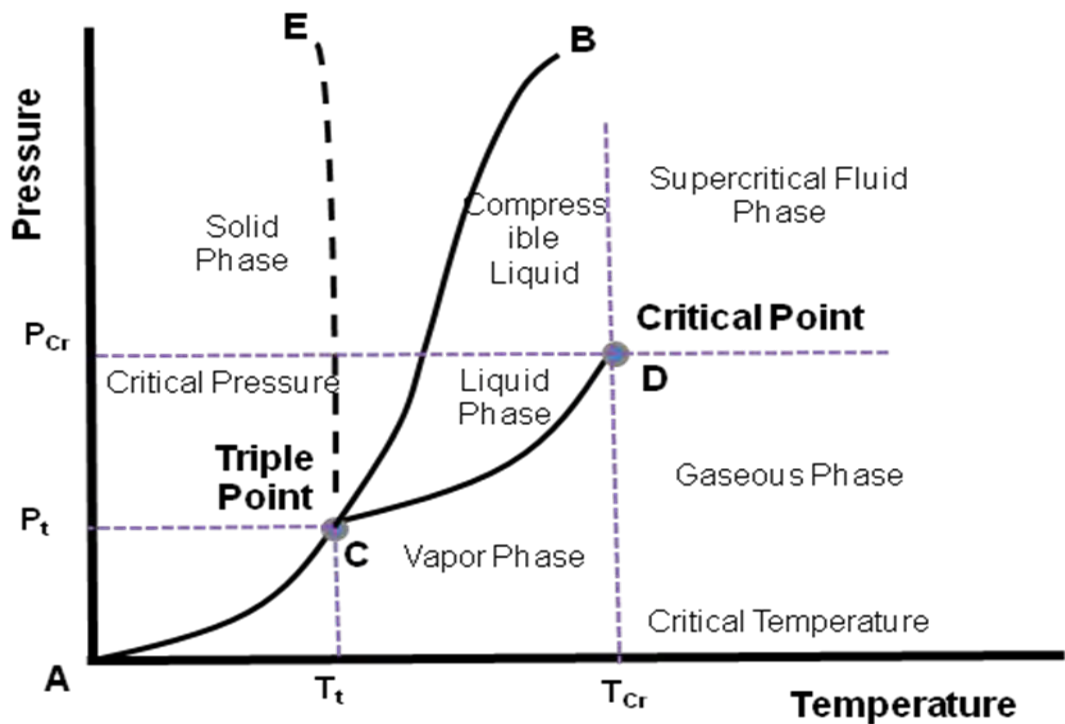
Figure 1.4; Comparison of Triple Point vs. Critical Point, a Graphical Perspective

Segment 1

Self-assessment Problems and Questions

1. A boiler is relocated from sea level to a location that is at an elevation of 10,000 ft MSL. Using the table below and Table 1.3, determine the temperature at which the water will boil if the boiler is assumed to be open to the atmosphere.

Altitude With Mean Sea Level as Ref.		Absolute Pressure in Hg Column		Absolute Atmospheric Pressure		
Feet	Meters	Inches Hg Column	mm Hg Column	psia	kg/cm²	kPa
0	0	29.9	765	14.7	1.03	101
500	152	29.4	751	14.4	1.01	99.5
1000	305	28.9	738	14.2	0.997	97.7
1500	457	28.3	724	13.9	0.979	96
2000	610	27.8	711	13.7	0.961	94.2
2500	762	27.3	698	13.4	0.943	92.5
3000	914	26.8	686	13.2	0.926	90.8
3500	1067	26.3	673	12.9	0.909	89.1
4000	1219	25.8	661	12.7	0.893	87.5
4500	1372	25.4	649	12.5	0.876	85.9
5000	1524	24.9	637	12.2	0.86	84.3
6000	1829	24	613	11.8	0.828	81.2
7000	2134	23.1	590	11.3	0.797	78.2
8000	2438	22.2	568	10.9	0.768	75.3
9000	2743	21.4	547	10.5	0.739	72.4
10000	3048	20.6	526	10.1	0.711	69.7
15000	4572	16.9	432	8.29	0.583	57.2
20000	6096	13.8	352	6.75	0.475	46.6

2. In problem (1), if the objective is just to heat the water close to the boiling point, will the boiler consume more or less fuel than it did when it was located at the sea level?

3. Answer the following questions for water at a temperature of 193°F and pressure of 10 psia:

a) Heat content for saturated water.

b) Specific heat (BTU/lbm) required to evaporate the water.
c) If the water were evaporated, what would the saturated vapor heat content be?

d) What state or phase would the water be in at the stated temperature and pressure?

e) What would the entropy of the water be while it is in saturated liquid phase?

f) What would the specific volume of the water be while it is in saturated vapor phase?

g) What would the phase of the water if the pressure is increased to 20 psia while keeping the temperature constant at 193°F?

Segment 2

Laws of Thermodynamics

Topics:
- Laws of thermodynamics and associated formulas
- Applications of laws of thermodynamics and associated case study

Introduction:

This segment explores major categories of thermodynamic systems based on their interaction with the surroundings or environment. The three major categories of thermodynamic systems are introduced and key differences between them are explained. Differentiation between the three types of thermodynamic systems is reinforced through tabular cross-referencing of characteristic properties.

Since open thermodynamic systems are somewhat more common than other type of thermodynamic systems, a detailed case study - involving a fossil fuel powered steam power generation system - is undertaken and explained in this segment. This case study highlights the practical significance and application of steam based thermodynamic systems. The detailed discussion and step by step analysis of each thermodynamic process involved provides the energy engineers an opportunity to understand and hone skills associated with thermodynamic analysis of steam based power harnessing and power generating systems.

The material in this segment and the case study build upon and utilize the thermodynamic concepts, principles, laws and computational methods covered in foregoing segments. For the reader's convenience - and to make the analysis versatile - the case study in this segment is presented in SI/metric units as well as US/imperial units.

Major Categories of Thermodynamic Systems:

Thermodynamic systems can be categorized in myriad ways. However, in this segment we will focus on categorization of thermodynamic systems based on their interaction with the surroundings or environment. From thermodynamic system and environment interface perspective, thermodynamic systems can be categorized as follows:

I. Open Thermodynamic Systems

II. Closed Thermodynamic Systems

III. Isolated Thermodynamic Systems

Open Thermodynamic Systems:

Open thermodynamic systems are systems in which, in addition to the exchange of heat energy with the surroundings, mass or matter are free to cross the system boundary. Also, in open thermodynamic systems, work is performed on or by the system. The type of open thermodynamic systems where entering mass flow rate is the same as the exiting mass flow rate is referred to as a *Steady Flow Open System*. Examples of Steady Flow Open Systems include pumps, compressors, turbines and heat exchangers.

Closed Thermodynamic Systems:

Closed thermodynamic systems are systems in which no mass crosses the system boundary. Energy, however, can cross through the system boundary in form of heat or work. Examples of closed thermodynamic systems include: sealed pneumatic pistons and refrigerant in a refrigeration system.

Isolated Thermodynamic Systems:

Isolated thermodynamic systems are systems in which *no* work is performed by or on the system; no heat is added or extracted from the system and no matter flows in or out of the system. Imagine a rigid sealed steel cylinder containing liquid nitrogen. This steel cylinder is heavily insulated and is

placed inside another sealed steel container such that the cylinder's walls do not come in contact with the outside steel container. If vacuum is now created between the outer container and the inner gas cylinder, you would have a thermodynamic system that is "isolated" for most practical purposes. In that, there would be negligible, in any, heat transfer between the liquid nitrogen and the environment or surroundings outside the outer sealed container. The liquid nitrogen is contained in a sealed steel container, with fixed volume; therefore, no work can be performed by the nitrogen (the system) or the environment (e.g. the air) outside the outer container. In addition, because of the containment or isolation attained through the steel cylinder and the outer sealed container, there would be no transfer of mass or matter.

Table 2.1 below facilitates comparison and cross-referencing of the three categories of thermodynamic systems. In addition, this table permits an examination of the three types of thermodynamic systems on the basis of three important thermodynamic process attributes, namely: (1) Mass Flow Across the Boundary, (2) Work Flow Across the Boundary, and (3) Heat Exchange with the Surroundings.

Interaction of Heat, Mass and Work in Thermodynamic Systems			
Type of Thermodynamic System	**Open**	**Closed**	**Isolated**
Mass Flow Across Boundary	Yes	No	No
Work Performed On or By the System	Yes	Yes	No
Heat Exchange with the Surroundings	Yes	Yes	No

Table 2.1 – Thermodynamic System Definition, Categorization and Cross-referencing

Laws of Thermodynamics:

Engineering discipline and study of science, in general, utilize principles and laws for developing or deriving equations. These equations are mathematical representation of the engineering or scientific principles and laws. Of course, the key purpose for deriving or developing equations is to be able to define or determine the value of unknown entities or unknown variables.

In our continued effort to expand our list of tools for thermodynamic system analyses, at this juncture, we will explore the first law of thermodynamics.

First Law of Thermodynamics Systems:

The first law of thermodynamics is a statement of the law of conservation of energy in the thermodynamics realm. In other words, the net energy entering a thermodynamic system is equal to the net change in the internal energy of the system plus the work performed by the system. All energy and work is accounted for at all points in the thermodynamic system. Of course, energy status at one point is compared with another point in the system as the laws of thermodynamics are applied to derive equations for system or process analysis.

As we transform the first law of conservation of energy into an equation with practical application and significance, it is important to reiterate that energy, heat and work are mathematically equivalent. Energy, heat and work can, therefore, be added or subtracted linearly in an equation.

Mathematical Statement of the **First Law of Thermodynamics** in a *Closed* Thermodynamic System is as follows:

$$Q = \Delta U + \Delta KE + \Delta PE + W \qquad \textbf{Eq. 2.1}$$

Where,

   ΔU = Change in Internal Energy
   ΔKE = Change in Kinetic Energy
   ΔPE = Change in Potential Energy
   Q = Heat energy entering (+), or leaving (-) the system

W = Work performed by the system on the surroundings is positive and work performed by the surroundings on the system is negative.

Mathematical Statement of the **First Law of Thermodynamics** in an *Open* Thermodynamic System, also referred to as SFEE, or Steady Flow Energy Equation, would be as follows:

$$Q = \Delta U + \Delta FE + \Delta KE + \Delta PE + W \qquad \text{Eq. 2.2}$$

Where,
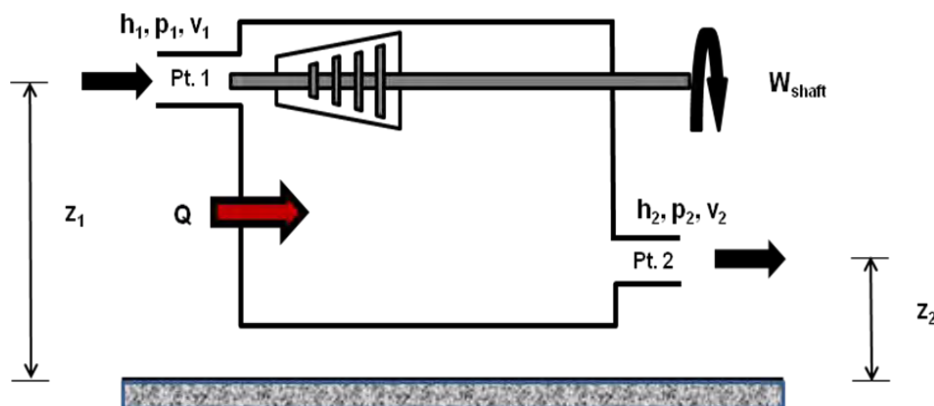
ΔU = Change in Internal Energy
ΔFE = Change in Flow Energy = $\Delta(PV)$
ΔKE = Change in Kinetic Energy
ΔPE = Change in Potential Energy
Q = Heat energy entering (+), or leaving (-) the system
W = Work performed by the system on the surroundings is positive and work performed by the surroundings on the system is negative.

Figure 2.1, Open thermodynamic system with a turbine.

The Steady Flow Energy Equation, or the SFEE, representing the application of the First Law of Thermodynamics in an Open Thermodynamic System can be written in the specific, or per unit mass, form as:

$$q = \Delta u + \Delta FE_{specific} + \Delta KE_{specific} + \Delta PE_{specific} + w_{shaft} \qquad \text{Eq. 2.3}$$

Since,

$$\Delta h = \Delta u + \Delta FE_{specific} \qquad \textbf{Eq. 2.4}$$

The SFEE can be written, in a more practical form, for a turbine open system as:

$$q = \Delta h + \Delta KE_{specific} + \Delta PE_{specific} + w_{shaft} \qquad \textbf{Eq. 2.5}$$

Or,

$$q = (h_2 - h_1) + 1/2 \cdot (v_2^2 - v_1^2) + g.(z_2 - z_1) + w_{shaft} \quad \textbf{Eq. 2.6}$$

This Steady Flow Energy Equation is stated in the SI or metric realm.

Where,

h_1 = Enthalpy of the steam entering the turbine, in kJ/kg.

h_2 = Enthalpy of the steam exiting the turbine, in kJ/kg.

v_1 = Velocity of the steam entering the turbine, in m/s.

v_2 = Velocity of the steam exiting the turbine, in m/s.

z_1 = Elevation of the steam entering the turbine, in meters.

z_2 = Elevation of the steam exiting the turbine, in meters.

q = Specific heat added or removed from the turbine system, in kJ/kg.

w_{shaft} = Specific work or work per unit mass; measured in kJ/kg.

W_{shaft} = Work performed by the turbine shaft, measured in kJ.

g = Acceleration due to gravity, 9.81 m/s^2

Since,

$$P_{shaft} = w_{shaft} \cdot \dot{m} \qquad \textbf{Eq. 2.7}$$

Where,

$\dot{m}$ = Mass flow rate of the system, in kg/sec or lbm/sec.

This SI version of the SFEE can be written, in a more useful form, for a power calculation in turbine type open system as**:**

$$\dot{m} \cdot (q) = \dot{m} \left[(h_2 - h_1) + 1/2 \cdot (v_2^2 - v_1^2) + g. (z_2 - z_1) \right] + \dot{m} \cdot w_{shaft} \qquad \textbf{Eq. 2.8}$$

Since,

$$\dot{Q} = \dot{m} \cdot (q) \qquad \textbf{Eq. 2.9}$$

Where, $\dot{\mathbf{Q}}$ = Flow rate of heat added or removed from the turbine system, in kJ/second.

The SI version of the SFEE can be written, in a more useful form, for a power calculation in turbine type open system as:

$$\dot{Q} = \dot{m}\left[(h_2 - h_1) + 1/2 \cdot (v_2^2 - v_1^2) + g \cdot (z_2 - z_1)\right] + P_{shaft} \qquad \textbf{Eq. 2.10}$$

For power computation purposes, the Open System Steady Flow Energy Equation can be rewritten, for application in the US or Imperial unit realm, as:

$$\dot{m} \cdot (q) = \dot{m}\left[(h_2 - h_1) + 1/2 \cdot (v_2^2 - v_1^2)/(g_c \cdot J) + g \cdot (z_2 - z_1)/(g_c \cdot J)\right] + \dot{m} \cdot w_{shaft} \qquad \textbf{Eq. 2.11}$$

Or,

$$\dot{Q} = \dot{m}\left[(h_2 - h_1) + 1/2 \cdot (v_2^2 - v_1^2)/(g_c \cdot J) + g \cdot (z_2 - z_1)/(g_c \cdot J)\right] + P_{shaft} \qquad \textbf{Eq. 2.12}$$

Where,

h_1 = Enthalpy of the steam entering the turbine, in BTU/lbm.

h_2 = Enthalpy of the steam exiting the turbine, in BTU/lbm.

v_1 = Velocity of the steam entering the turbine, in ft/s.

v_2 = Velocity of the steam exiting the turbine, in ft/s.

z_1 = Elevation of the steam entering the turbine, in ft.

z_2 = Elevation of the steam exiting the turbine, in ft.

q = Specific heat added or removed from the turbine system, in BTU/lbm

$\dot{Q}$ = Flow rate of heat added or removed from the turbine system, in BTUs/sec.

w_{shaft} = Specific work or work per unit mass; measured in BTU/lbm.

W_{shaft} = Work performed by the turbine shaft, measured in BTUs.

$\dot{m}$ = Mass flow rate of the system, in lbm/sec.

g = Acceleration due to gravity, 32.2 ft/s^2

g_c = Gravitational constant, 32.2 lbm-ft/lbf-sec^2

J = 778 ft-lbf/BTU

Second Law of Thermodynamics:

The Second Law of Thermodynamics can be stated in multiple ways. Some of the more common and practical statements of the Second Law of Thermodynamics are, briefly, discussed in this section

The Second Law of Thermodynamics is also known as the law of increasing entropy. While quantity of total energy remains constant in the universe as stipulated by the First Law of Thermodynamics, the Second Law of Thermodynamics states that the amount of usable, work producing, energy in the universe continues to decline; irretrievably lost in the form of unusable energy.

Since entropy is defined as unusable energy, the Second Law of Thermodynamics can be interpreted to state that in a closed system, such as the universe, entropy continues to increase. In other words, the second law of thermodynamics states that the net entropy must always increase in practical, irreversible cyclical processes.

The second law of thermodynamics can also be stated mathematically in the form of Eq. 2.13, below:

$$\Delta S \geq \int \frac{\delta Q}{T}$$
Eq. 2.13

Equation 2.13 stipulates that the increase in entropy of a thermodynamic system must be greater than or equal to the integral of the incremental heat absorbed, divided by the temperature during each incremental heat absorption.

Kevin-Planck statement of the second law of thermodynamics, effectively, implies that it is impossible to build a cyclical engine that has an efficiency of 100%.

Case Study 2.1, SI Units:

Technical feasibility of a toping cycle cogenerating power plant is being studied at Station "Zebra." This facility is to be stationed in a remote Arctic region. The objective of this plant is to produce steam and generate electricity for an Arctic Environmental Monitoring and Deep Sea Mining Facility. Due to saltwater corrosion risk, it has been established that local glacier ice will be harvested and utilized for steam production purposes. A natural gas boiler is to be used to generate steam. The average temperature of the glacier ice hovers at -10 °C, through the year. The glacier ice is to be melted and converted to 500 °C, 2.5 MPa, steam; the steam enters the turbine at this temperature and pressure. The steam is discharged by the turbines at 150 °C and 50 kPa. This discharged steam is used for the mining process and to heat the station. The condensate is used as potable, utility and process water. Assume that the turbine represents an open, steady flow, thermodynamic system. In other words, the SFEE, Steady Flow Energy Equation applies. Also assume the potable water demand and flow, along the thermodynamic process stages to be relatively negligible. Station Zebra would operate on a 24/7 schedule. For simplicity, the thermodynamic process flow for this system is laid out in Figure 2.2. **Note:** In an actual project setting, developing such a process flow diagram would constitute the first order of business as an energy engineer begins analyzing this process.

a) Estimate the mass flow rate **ṁ** for generation of 10 MW electrical power if the rate of turbine casing heat loss, **$\dot{Q}$** , is 30 kJ/s (or 0.03 MW), exit velocity of steam, **v_2** , is 35 m/s, entrance velocity of steam, **v_1** , is 15 m/s, steam exit elevation is **z_2** = 1m, steam intake elevation is **z_1** = 0.5m. Assume electric power generator efficiency of 90%. Extrapolate the answer into approximate truck loads per hour. Assume truck capacity of approximately 10 cubic meters.

b) Assume that the power station is generating 10 MW of electric power. Calculate the amount of total heat energy needed, in BTU's/hr, to convert -10°C harvested ice to 500°C, 2.5 MPa steam per hour.

c) Calculate the volume, in cu-ft, of natural gas required to power up the station, each day. Assume 98% burner efficiency.

d) If the natural gas transportation cost is **$4.85/DT** in addition to the well head or commodity cost stated in Table 2.1, what would be the annual fuel cost of operating this station?

e) What is the overall energy efficiency of the power station?

f) If heat is added to the steam turbines, would the steady flow energy process in the turbine system constitute an adiabatic process or a non-adiabatic process?

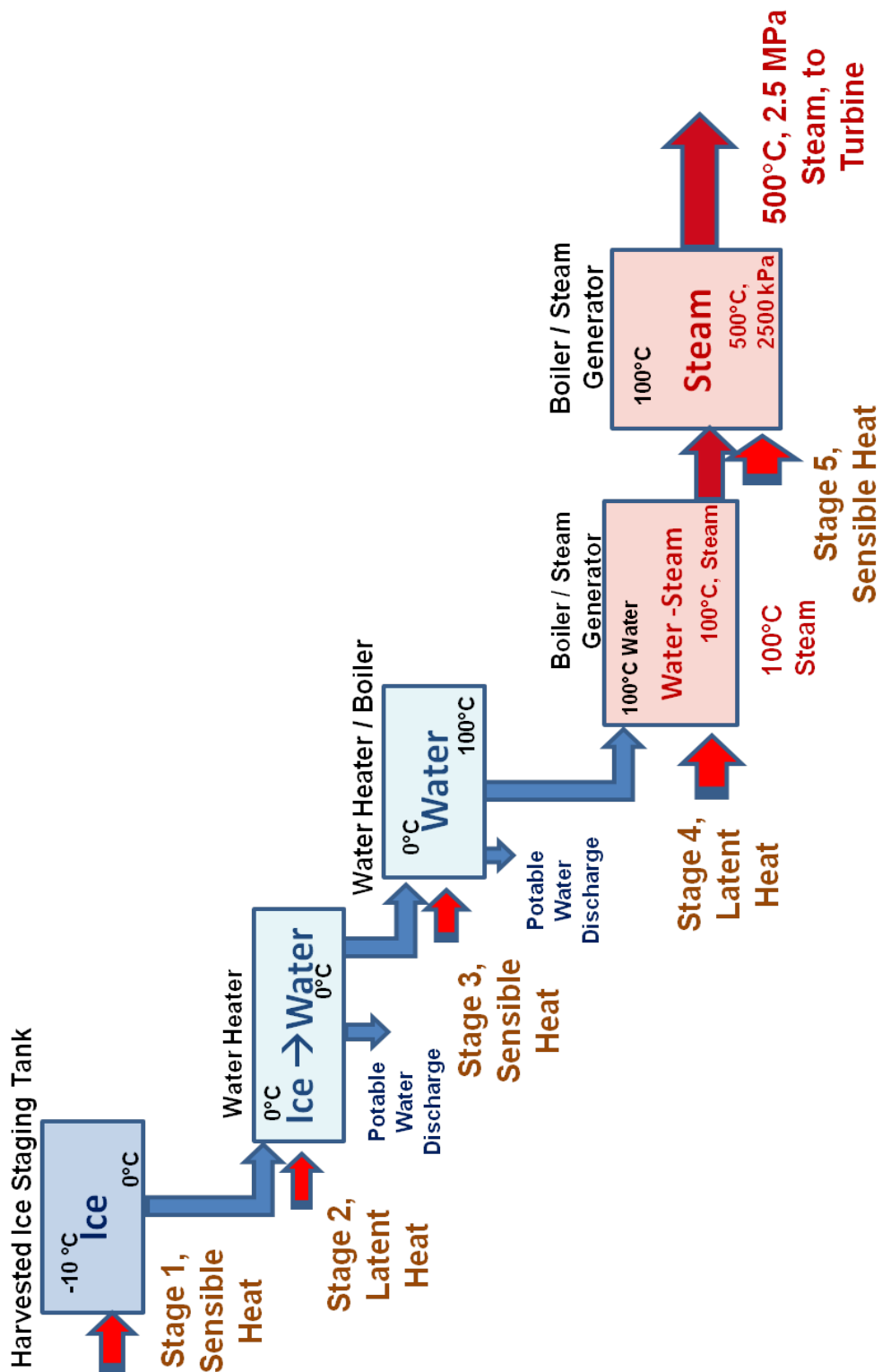g) What is the change in entropy, **Δs**, in the turbine system?

Figure 2.2, Thermodynamic Process Flow Diagram, Case Study 2.1

Energy Content and Cost Comparison

Fuel Type or Energy Source	Heat Content					Approx. Cost		BTU/kWh	US Sept. 2010 Average $/kWh	$/MMBTU
	DT/gal	BTU/gal	MMBTU/Barrel	BTU/Cu-ft.	DT/MCF	Price Per Gallon*	$/DT*			
Propane	0.092	91,600	3.35	2,488		$1.42	$15.38			27.29
Diesel/No. 2 Fuel Oil	0.138	140,000	5.6			$2.65	$19.17			23.429
No. 6 Fuel Oil	0.144	143,888	6.8							
Natural Gas				1,034	1.034	$1.00	$4.15			4.15
Electricity							$30.01	3,412	0.1024	30.01

* Note: These costs represent a January 3, 2011 snapshot of wholesale or industrial market costs.

Table 2.1, Energy Content and Cost Comparison of Common Fuels

Case Study Solution Strategy

Before embarking on the analyses and solution for this case study, let's highlight some important facts from the case study statement.

1) As apparent from the case study statement, the working fluid or system consists of water, in various phases.

2) Water is introduced into the overall thermodynamic system in form of -10°C ice and is then heated during various sensible and latent stages. It is, finally, fed into the turbine as 500°C, 2.5 MPa superheated steam.

3) Unlike a typical Rankin Cycle Heat Engine, the steam exhausted from the turbine is not condensed, pumped and recycled through the boilers to repeat the heat cycle. Instead, fresh ice is harvested, melted and introduced as working fluid.

4) Even though Figure 2.2 shows potable water being removed from the system at Stages 2 and 3, it is assumed to be negligible. Thus supporting the assumption that mass flow rate of the working fluid stays constant through the system.

5) Fuel heat content and fuel cost information is available through Table 2.1.

6) Specific heat and latent heat data for water and ice are provided through Tables 2.2 and 2.3, for sensible heat and latent heat calculations, respectively.

7) Most of the data pertinent to the application of SFEE equation, at the turbine, is given, including the velocities, temperatures, pressures and elevations.

8) Final output of the power generating station is given in terms of the 10 MW rating of the generator and its stated efficiency of 90%.

The overall thermodynamic process flow can be tiered into stages that involve either sensible heating or latent heating. All heating stages of this comprehensive process are depicted in Figure 2.2. Each stage of the overall process is labeled with pertinent entry and exit temperature and pressure, as applicable. Each stage is named as either Sensible or Latent Stage. Furthermore, each stage states the phase of water at point of entry and exit.

Approximate Specific Heat, c_p, for Selected Liquids and Solids			
Substance	c_p in J/gm K	c_p in cal/gm K or BTU/lb F	Molar C_p J/mol K
Aluminum	0.9	0.215	24.3
Bismuth	0.123	0.0294	25.7
Copper	0.386	0.0923	24.5
Brass	0.38	0.092	...
Gold	0.126	0.0301	25.6
Lead	0.128	0.0305	26.4
Silver	0.233	0.0558	24.9
Tungsten	0.134	0.0321	24.8
Zinc	0.387	0.0925	25.2
Mercury	0.14	0.033	28.3
Alcohol(ethyl)	2.4	0.58	111
Water	**4.186**	**1**	**75.2**
Ice (-10 C)	**2.05**	**0.49**	**36.9**
Granite	0.79	0.19	**...**
Glass	0.84	0.2	

Table 2.2, Specific Heat, c_p, for Selected Liquids and Solids

	Latent Heat Fusion h_{sl}	**Latent Heat Sublimation** h_{ig}	**Latent Heat Vaporization** h_{fg}
kJ/kg	333.5	2838	2257
BTU/lbm	143.4	1220	970.3
kcal/kg	79.7	677.8	539.1

Table 2.3, Latent Heat for Phase Transformation of Water

a) Estimate the mass flow rate $\dot{m}$ for generation of 10 MW electrical power output if the rate of turbine casing heat loss, $\dot{Q}$, is 30 kJ/s (or 0.03 MW), exit velocity of steam, v_2 , is 35 m/s, entrance velocity of steam, v_1 , is 15 m/s, steam exit elevation is $z_2 = 1$m, steam intake elevation is $z_1 = 0.5$m. Assume electric power generator efficiency of 90%. Extrapolate the answer into approximate truck loads per hour. Assume truck capacity of approximately 10 cubic meters.

Solution:
This part of the case study can be analyzed and solved by simply focusing on the very last stage and applying the SFEE in form of Eq. 2.10. The computation of mass flow rate does not require assessment of the heat required at the various stages of the overall thermodynamic process, in this case study, because of the following key assumption included in the problem statement:

"Also assume the potable water demand and flow, along the thermodynamic process stages to be relatively negligible. "

In other words the mass flow rate is assumed to be constant through out the process, and any discharge of water during individual stages of the overall process is negligible.

The turbine segment of the overall power generating system is illustrated in Figure 2.3, below. The enthalpy values are obtained from the superheated steam table excerpt in Table 2.4.
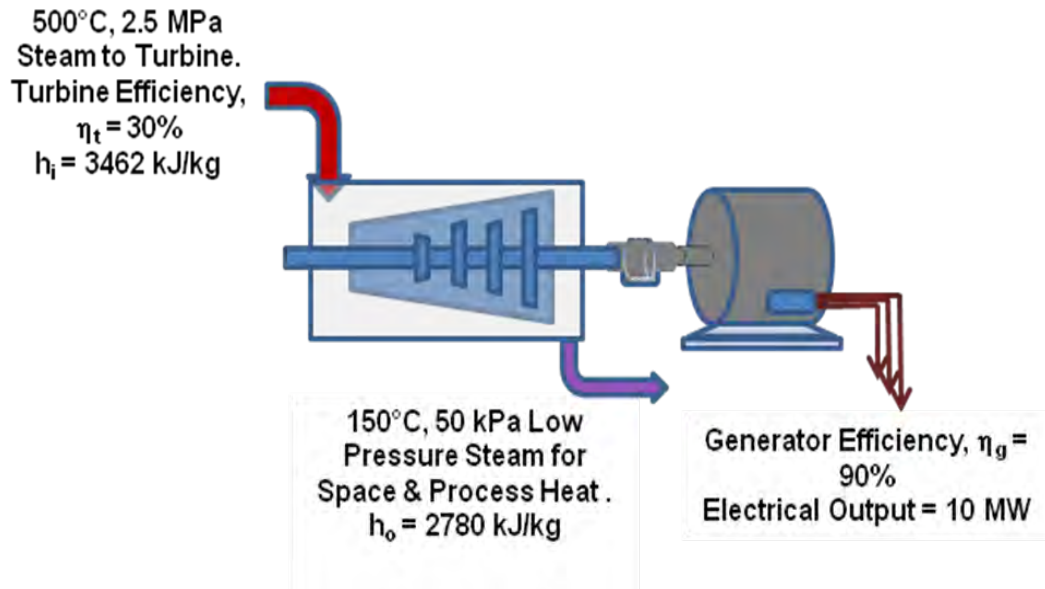
Figure 2.3, Case Study 2.1, Mass Flow Rate Analysis

Apply SFEE, in form of **Eq. 2.10**.

$$\dot{Q} = \dot{m}\,[(h_2 - h_1) + 1/2 \cdot (v_2{}^2 - v_1{}^2) + g.(z_2 - z_1)] + P_{shaft} \qquad \textbf{Eq. 2.10}$$

Rearrangement of Eq. 2.10 yields:

$$\dot{m} = (\dot{Q} \; - \; P_{shaft}) / [(h_2 - h_1) + 1/2 \cdot (v_2{}^2 - v_1{}^2) + g.(z_2 - z_1)]$$

Given:

$\dot{Q} = -0.03 \text{ MW} = -0.03 \times 10^6 \text{ W} = -0.03 \times 10^6 \text{ J/s}$

$P_{shaft} = (10 \text{ MW}) / \eta_g$
   $= (10 \text{ MW}) / (0.9)$
   $= 11.11 \times 10^6 \text{ W}$
   $= 11.11 \times 10^6 \text{ J/s}$

$h_2 = h_o = 2780 \text{ kJ/kg} = 2780 \times 10^3 \text{ J/kg}$ {See Table 2.4}

$h_1 = h_i = 3462 \text{ kJ/kg} = 3462 \times 10^3 \text{ J/kg}$ {See Table 2.4}

$v_2 = 35 \text{ m/s}$

v_1 = 15 m/s
z_2 = 1m
z_1 = 0.5m

Apply Eq. 2.10, in its rearranged form as follows:

$$\dot{m} = (\dot{Q} - P_{shaft}) / [(h_2 - h_1) + 1/2 \cdot (v_2^2 - v_1^2) + g.(z_2 - z_1)]$$

$$\dot{m} = (-0.03 \times 10^6 \text{ J/s} - 11.11 \times 10^6 \text{ J/s}) / [(2780 \times 10^3 \text{ J/kg} - 3462 \times 10^3 \text{ J/kg}) + 1/2 \{(35m/s)^2 - (15m/s)^2\} + 9.81 \text{ m/s}^2.(1m - 0.5m)]$$

$$\dot{m} = (-0.03 \times 10^6 \text{ J/s} - 11.11 \times 10^6 \text{ J/s}) / (-682,000 \text{ J/kg} + 500 \text{ J/kg} + 4.9 \text{J/kg})$$

$\dot{m}$ = 16.35 kg/sec ,

Or,

$\dot{m}$ = {(16.35 kg/907.2kg/ton)/sec }.(3600 sec/hour)
$\dot{m}$ = 65 tons/hour, or

Since the Density of ice = 916.8 kg/m^3,

The Volumetric flow rate, $\dot{v}$ **,** would be:

$$\dot{v} = (16.35 \text{ kg/sec}).(3600 \text{ sec/hr})/(916.8 \text{ kg/m}^3)$$

$$\dot{v} = 64.19 \text{ cu-meters /hr}$$

At an estimated 10 cubic meters per truck load, this volumetric mass flow rate would amount to:

$$\dot{v} = (64.19 \text{ cu-meters /hr})/10$$

Or,

$$\dot{v} = 6.4 \text{ truck loads per hour}$$

Properties of Superheated Steam Metric/SI Units						
Abs. Press. in MPa		Temp. in °C	"v" in m³/kg	"h" in kJ/kg	"s" in kJ/kg.°K	
(Sat. T, °C)		150	300	500	650	800
0.05	v	3.889	5.284	7.134		
(81.33)	h	2780.1	3075.5	3488.7		
	s	7.9401	8.5373	9.1546		
0.1	v	1.9367	2.6398	3.5656		
(99.61)	h	2776.6	3074.5	3488.1		
	s	7.6147	8.2171	8.8361		
1.0	v		0.2580	0.3541	0.4245	0.4944
(179.89)	h		3051.7	3479.0	3810.5	4156.2
	s		7.1247	7.7640	8.1557	8.5024
2.5	v		0.0989	0.13998	0.1623	0.1896
(223.99)	h		3008.8	3462.1	3799.7	4148.9
	s		6.6438	7.3234	7.7056	8.0559
3.0	v		0.0812	0.1162	0.1405	0.1642
(233.86)	h		2994.3	3457.0	3797.0	4147.0
	s		6.5412	7.2356	7.6373	7.9885
4.0	v		0.0589	0.0864	0.1049	0.1229
(250.36)	h		2961.7	3445.8	3790.2	4142.5
	s		6.3638	7.0919	7.4989	7.8523

Table 2.4. Excerpt, Superheated Steam Table, SI Units.

b) Assume that the power station is generating 10 MW of electric power. Calculate the amount of heat needed, in BTU's/hr, to convert -10 °C harvested ice to 500 °C, 2.5 MPa steam, per hour.

Solution:

Part (b) of this case study does require accounting for heat added during each of the five (5) stages of the overall process. Therefore, this part is divided into five sub-parts, each involving either sensible or latent heat calculation, based on the entry and exit temperature and phase status.

Table 2.2 lists specific heat for water and ice. These heat values will be used in the sensible heat calculations associated with Part (b). Table 2.3 lists latent heat values for water. These values will be used to compute the latent heats associated with stages that involve phase transformation.

(i) Calculate the heat required to heat the ice from -10°C to 0°C:
Since there is no change in phase involved, the entire heat absorbed by the ice (working substance) in this stage would be sensible heat.

The first stage of the overall power generating system is illustrated in Figure 2.4, below.
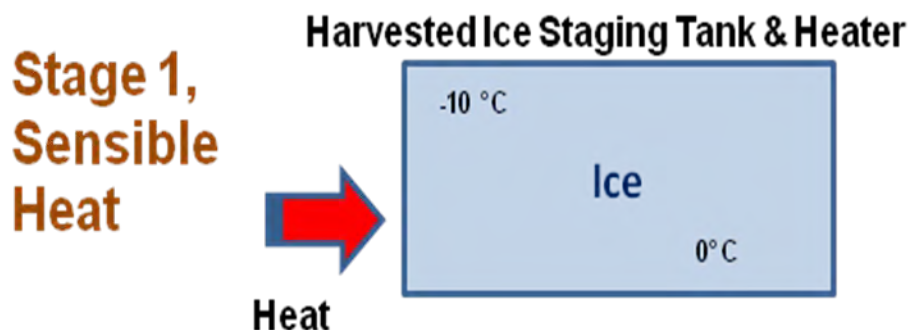
Figure 2.4, Case Study 2.1 Stage 1 Sensible Heat Calculation

Given:
$T_i = $ **-10 °C**
$T_f = $ **0 °C**
$c_{ice} = $ **2.05 kJ/kg. °K**   {Table 2.3}

Utilizing the given information:

$$\Delta T = T_f - T_i$$
$$\therefore \Delta T = 0 - (-10 \ °C)$$
$$= +10 \ °C$$

Since **ΔT** represents the *change* in temperature and not a specific absolute temperature,

$$\therefore \Delta T = +10 \ °C = +10 \ °K$$

The mathematical relationship between sensible heat, mass of the working substance, specific heat of the working substance and change in temperature can be stated as:

$$Q_{s(heat \ ice)} = m \ . \ c_{ice} \ . \ \Delta T \qquad\qquad\qquad \textbf{Eq. 2.14}$$

And,

$$\dot{Q}_{s(heat \ ice)} = \dot{m} \ . \ c_{ice} \ . \ \Delta T \qquad\qquad\qquad \textbf{Eq. 2.15}$$

Where,

$Q_{s(heat \ ice)}$ = Sensible heat required to heat the ice over ΔT
$\dot{Q}_{s(heat \ ice)}$ = Sensible heat flow rate required to heat the ice over ΔT
m = Mass of ice being heated
c_{ice} = Specific heat of ice = **2.05 kJ/kg. °K**
ΔT = Change in temperature, in **°C or °K**
$\dot{m}$ = Mass flow rate of water/ice
= **16.35 kg/sec**

Or,

$\dot{m}$ = (16.35 kg/sec) . (3600 sec/hr)
= **58,860 kg/hr**

Then, by application of **Eq. 2.15**:

$$\dot{Q}_{s(heat \ ice)} = (58,860 \ kg/hr) \ . \ (2.05 \ kJ/kg. \ °K) \ . \ (10 \ °K)$$

Or,

$$\dot{Q}_{s(heat \ ice)} = \textbf{1,206,630 kJ/hr}$$

Since there are 1.055 kJ per BTU,

$$\dot{Q}_{s(heat\ ice)} = (1,206,630\ kJ/hr)/(1.055\ kJ/BTU)$$

Or,

$$\dot{Q}_{s(heat\ ice)} = 1,143,725\ BTU/hr$$

(ii) Calculate the heat required to melt the ice at 0°C.

Since change in phase *is* involved in this case, the heat absorbed by the ice (working substance) in this stage would be latent heat.

The 2nd stage of the overall power generating system is illustrated in Figure 2.5, below.
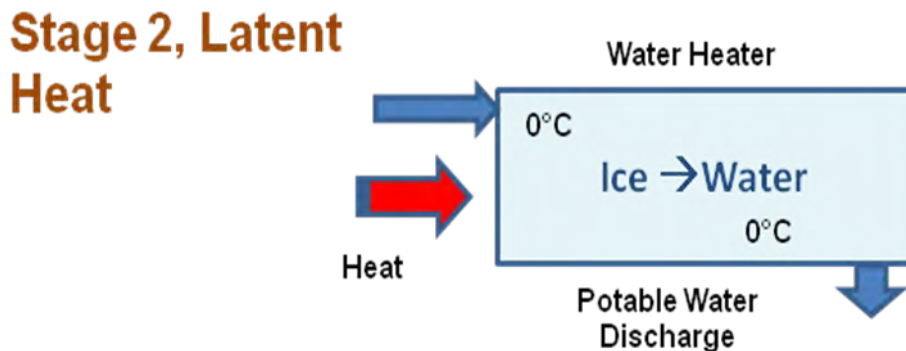
Figure 2.5, Case Study 2.1 Stage 2 Latent Heat Calculation

The mathematical relationship between latent heat, mass of the working substance, and the heat of fusion of ice can be stated as:

$$Q_{l(latent\ ice)} = h_{sl\ (ice)} \cdot m \qquad\qquad \textbf{Eq. 2.16}$$

And,

$$\dot{Q}_{l(latent\ ice)} = h_{sl\ (ice)} \cdot \dot{m} \qquad\qquad \textbf{Eq. 2.17}$$

Where,

$Q_{l(latent\ ice)}$ = Latent heat required to melt a specific mass of ice, isothermally

$\dot{Q}_{l(latent\ ice)}$ = Latent heat flow rate required to melt a specific mass of ice, isothermally, over a period of time

m = Mass of ice being melted

$\dot{m}$ = Mass flow rate of water/ice

= 60 tons/hr

$$= (60 \text{ tons/hr}) \cdot (907.2 \text{ kg/ton})$$
$$= \textbf{58,860 kg/hr,} \text{ same as part (a) (i)}$$

$h_{sl \text{ (ice)}}$ = Heat of fusion for Ice

$$= \textbf{333.5 kJ/kg} \qquad\qquad \{\text{Table 2.3}\}$$

Then, by application of **Eq. 2.17**:

$$\dot{Q}_{\text{l(latent ice)}} = \textbf{h}_{sl \text{ (ice)}} \cdot \dot{\textbf{m}}$$

$$\dot{Q}_{\text{l(latent ice)}} = (333.5 \text{ kJ/kg}) \cdot (58,860 \text{ kg/hr})$$

$$\dot{Q}_{\text{l(latent ice)}} = \textbf{19,629.810 kJ/hr}$$

Since there are 1.055 kJ per BTU,

$$\dot{Q}_{\text{l(latent ice)}} = \textbf{(19,629,810 kJ/hr)/(1.055 kJ/BTU)}$$

Or,

$$\dot{Q}_{\text{l(latent ice)}} = \textbf{18,606,455 BTU/hr}$$

Note that the specific heat required to melt ice is called heat of fusion because of the fact that the water molecules come closer together as heat is added in the melting process. The water molecules are held apart at specific distances in the crystallographic structure of solid ice. The heat of fusion allows the molecules to overcome the crystallographic forces and "**fuse**" to form liquid water. This also explains why the density of water is higher than the density of ice.

(iii) Calculate the heat reqd. to heat the water from 0°C to 100°C:
The 3$^{\text{rd}}$ stage of the overall power generating system is illustrated in Figure 2.6, below. Since no phase change is involved in this stage, the heat absorbed by the water in this stage would be sensible heat.
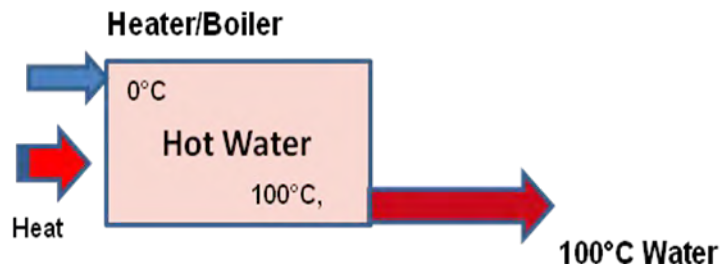
Figure 2.6, Case Study 2.1 Stage 3 Sensible Heat Calculation

Given:

$T_i = 0\ °C$

$T_f = 100\ °C$

$c_{p\text{-water}} = 4.19\ kJ/kg.\ °K$ {Table 2.3}

Utilizing the given information:

$\Delta T = T_f - T_i$

$\therefore \Delta T = 100\ °C – 0°C$

$= 100\ °C$

Since **ΔT** represents the *change* in temperature and not a specific absolute temperature,

$\therefore \Delta T = 100\ °K$

The mathematical relationship between sensible heat, mass of the working substance, specific heat of water (the working substance), and change in temperature can be stated as:

$Q_{s(water)} = m \cdot c_{p\text{-water}} \cdot \Delta T$ **Eq. 2.18**

And,

$\dot{Q}_{s(water)} = \dot{m} \cdot c_{p\text{-water}} \cdot \Delta T$ **Eq. 2.19**

Where,

$Q_{s(water)}$ = Sensible heat required to heat the water over ΔT

$\dot{Q}_{s(water)}$ = Sensible heat *flow rate* required to heat the water over ΔT

m = Mass of water being heated

$c_{p\text{-water}}$ = Specific heat of water = **4.19 kJ/kg. °K**

$\dot{m}$ = Mass flow rate of water = **58,860 kg/hr**, as calculated in part (a)

$\mathbf{\Delta T}$ = Change in temperature, in $\mathbf{°C\ or\ °K}$

Then, by applying **Eq. 2.19**:

$\dot{\mathbf{Q}}_{\mathbf{s(water)}} = \dot{\mathbf{m}} \mathbf{.\ c_{p\text{-}water}\ .\ \Delta T}$

$\dot{\mathbf{Q}}_{\mathbf{s(water)}} = (58{,}860 \text{ kg/hr}) \mathbf{.}\ (4.19 \text{ kJ/kg. °K}) \mathbf{.}\ (100 \text{ °K})$

$\dot{\mathbf{Q}}_{\mathbf{s(water)}} = \mathbf{24{,}662{,}340 \text{ kJ/hr}}$

Since there are 1.055 kJ per BTU,

$\dot{\mathbf{Q}}_{\mathbf{s(water)}} = \mathbf{(24{,}662{,}340 \text{ kJ/hr})/(1.055 \text{ kJ/BTU})}$

Or,

$\dot{\mathbf{Q}}_{\mathbf{s(water)}} = \mathbf{23{,}376{,}626 \text{ BTU/hr}}$

(iv) Calculate the heat required to convert 100°C water to 100°C steam:

The 4th stage of the overall power generating system is illustrated in Figure 2.7, below. Since change in phase *is* involved in this case, the heat absorbed by the water in this stage would be latent heat.
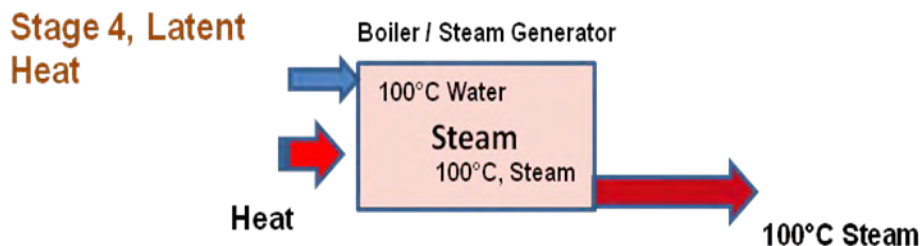
Figure 2.7, Case Study 2.1 Stage 4 Latent Heat Calculation

The mathematical relationship between latent heat of vaporization for water, $\mathbf{h_{fg(water)}}$, mass of the water, and the total heat of vaporization of water, $\mathbf{Q_{l(latent\ water)}}$, can be stated as:

$\mathbf{Q_{l(latent\ water)} = h_{fg\ (water)}\ .\ m}$ **Eq. 2.20**

And,

$\dot{\mathbf{Q}}_{\mathbf{l(latent\ water)}} = \mathbf{h_{fg\ (water)}\ .\ \dot{m}}$ **Eq. 2.21**

Where,

$Q_{l(latent\ water)}$ = Latent heat of vaporization of water required to evaporate a specific mass of water, isothermally

$\dot{Q}_{l(latent\ water)}$ = Latent heat of vaporization *flow rate* required to evaporate a specific mass of water, isothermally, over a given period of time

m = Mass of water being evaporated

$\dot{m}$ = Mass flow rate of water

= 60 tons/hr

= (60 tons/hr) . (907.2 kg/ton)

= **58,860 kg/hr ,** same as part (a) (i)

$h_{fg\ (water)}$ = latent heat of vaporization for water

= **2257 kJ/kg**   {From the steam tables and Table 2.3}

Then, by application of **Eq. 2.21**:

$$\dot{Q}_{l(latent\ water)} = h_{fg\ (water)}\ .\ \dot{m}$$

$$\dot{Q}_{l(latent\ water)} = (2257\ kJ/kg).\ (58,860\ kg/hr)$$

$$\dot{Q}_{l(latent\ water)} = \textbf{132,487,020 kJ/hr}$$

Since there are 1.055 kJ per BTU,

$$\dot{Q}_{l(latent\ water)} = \textbf{(132,487,020 kJ/hr)/(1.055 kJ/BTU)}$$

Or,

$$\dot{Q}_{l(latent\ water)} = \textbf{125,921,346 BTU/hr}$$

(v) Calculate the heat reqd. to heat the steam from 100 °C, 1-atm (102 KPa, or 1-bar) to 500 °C, 2.5 MPa superheated steam:

The 5th stage of the overall power generating system is illustrated in Figure 2.8, below. Since this stage involves no phase change, the heat absorbed by the steam is sensible heat.

In superheated steam phase, the heat required to raise the temperature and pressure of the steam can be determined using the enthalpy difference between the initial and final conditions.
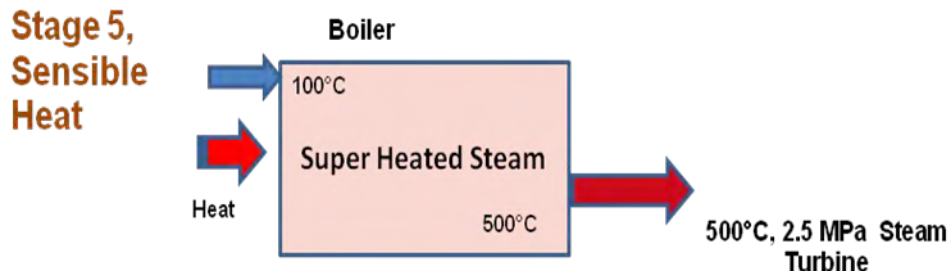
Figure 2.8, Case Study 2.1 Stage 5 Sensible Heat Calculation

Given:

T_i = 100 °C

P_i = 1- Atm. Note: At 100 °C, the saturation pressure is 1- Atm, 1- Bar, or 102 kPa

T_f = 500 °C

P_f = 2.5 MPa

For the initial and final temperature and pressure conditions stated above, the enthalpy values, as read from saturated steam table excerpt in Table 2.4a and the superheated steam table excerpt in Table 2.4, are as follows:

h_i = 2676 kJ/kg at 100 °C, 1-Atm

h_f = 3462 kJ/kg at 500 °C, 2.5 MPa

Properties of Saturated Steam By Temperature

Metric/SI Units

Temp. °C	Abs. Press. MPa	Specific Volume m³/kg		Enthalpy kJ/kg			Entropy kJ/kg		Temp. °C
		Sat. Liquid v_L	Sat. Vapor v_V	Sat. Liquid h_L	Evap. h_{fg}	Sat. Vapor h_V	Sat. Liquid s_L	Sat. Vapor s_V	
20	0.002339	0.0010018	57.7610	83.920	2454.1	2537.5	0.2965	8.6661	20
50	0.012351	0.0010121	12.0280	209.34	2382.7	2591.3	0.7038	8.0749	50
100	0.101420	0.0010435	1.6719	419.10	2257.0	2675.6	1.3070	7.3541	100
200	1.554700	0.0011565	0.1272	852.39	1940.7	2792.1	2.3308	6.4303	200

Table 2.4a. Excerpt, Saturated Steam Table, SI.

Equations for determining the heat required to boost the steam from **100 °C, 1-Atm** to **500 °C, 2.5 MPa** are as follows:

$$\Delta Q_{steam} = (h_f - h_i) \cdot m \qquad\qquad \text{Eq. 2.22}$$

$$\dot{Q}_{steam} = (h_f - h_i) \cdot \dot{m} \qquad\qquad \text{Eq. 2.23}$$

Where,

ΔQ_{steam} = Addition of heat required for a specific change in enthalpy
$\dot{Q}_{steam}$ = Rate of addition of heat for a specific change in enthalpy
h_i = Initial enthalpy
h_f = Final enthalpy
m = Mass of steam being heated
$\dot{m}$ = Mass flow rate of steam as calculated in part (a) of this case study
   = **58,860 kg/hr**   {From Part (a)}

Then, by applying **Eq. 2.23**:

$$\dot{Q}_{steam} = (h_f - h_i) \cdot \dot{m}$$

$$\dot{Q}_{steam} = (3462 \text{ kJ/kg} - 2676 \text{ kJ/kg}) \cdot (58,860 \text{ kg/hr})$$

$$\dot{Q}_{steam} = \textbf{46,263,960 kJ/hr}$$

Since there are 1.055 kJ per BTU,

$$\dot{Q}_{s(water)} = \textbf{(46,263,960kJ/hr)/(1.055 kJ/BTU)}$$

Or,

$$\dot{Q}_{s(water)} = \textbf{43,852,095 BTU/hr}$$

After assessing the heat added, per hour, during each of the five (5) stages of the steam generation process, add all of the heat addition rates to compile the total heat addition rate for the power generating station. The tallying of total heat is performed in BTU's/hr as well as kJ/hr.

Total Heat Addition Rate in kJ/hr:

Total Heat Required to Generate 500 °C, 2.5 MPa steam from -10 °C Ice, at 58,860 kg/hr

     = 1,206,630 kJ/hr + 19,629.810 kJ/hr + 24,662,340 kJ/hr +
     132,487,020 kJ/hr + 46,263,960 kJ/hr
     = 224,609,760 kJ/hr

 Total Heat Addition Rate in BTU's/hr:

Total Heat Required to Generate 500 °C, 2.5 MPa steam from -10 °C Ice, at 58,860 kg/hr

     = 1,143,725 BTU/hr + 18,606,455 BTU/hr + 23,376,626 BTU/hr +
     125,580,114 BTU/hr + 43,852,095 BTU/hr

     = 212,559,014 BTU/h

c) Calculate the volume, in cu-ft, of natural gas required to power up the station, each day. Assume 98% burner efficiency.

Solution:

This part of Case Study 2.1 requires computation of the amount (volume) of natural gas required to power up the station each day. This calculation is straight forward after the derivation of the total energy required, per hour, in part (b). However, the hourly energy requirement must be "scaled up" to account for the 98% efficiency of the boiler burner.

The hourly energy requirement, in kJ or BTU's, can be extended into daily usage. The daily energy usage can then be converted into the volume of natural gas required, based on natural gas energy content listed in Table 2.1.

Total Energy Required Per Day = (212,559,014 BTU/hr)/(0.98) x 24 hr
                                 = 5,205,526, 873 BTU's

Since Natural Gas Energy Content = 1034 BTU/cu-ft,

Total volume of natural gas required per day

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$$= (5,205,526, 873 \text{ BTU's}) / 1034 \text{ BTU/cu-ft}$$
$$= \textbf{5,034,358 cu-ft}$$

d) If the natural gas transportation cost is $ 4.85/DT in addition to the well head or commodity cost stated in Table 2.1, what would be the annual fuel cost of operating this station?

Solution:

Part (d) of the case study relates to the computation of total annual cost of fuel for the power generation station. This requires the extrapolation of daily energy consumption into *annual* energy consumption. The annual energy consumption is then multiplied with the *total*, delivered, cost rate in **$/DT** to obtain the annual cost in dollars.

From part (b), the heat energy required for operating the power station = **212,559,014** BTU/hr

Then, based on 24/7 operating schedule assumption, the total energy required per year would be:

= (212,559,014 BTU/hr)/(0.98) x 8760 hrs/year
= 1.900 E+12 BTU's

Since there are 1,000,000 BTU's, or 1 MMBTU, per DT, the Total Annual Energy Consumption in DT would be:

= **1.900 E+12 BTU's** / (1,000,000 BTU's/DT)
= 1,900,017 DT's

The total cost of natural gas, per DT =
Cost at the Source/Well + Transportation Cost
$$= \$4.15/DT + \$4.85/DT$$
$$= \textbf{\$9.00/DT}$$

Then, the Total Annual Cost for Producing 10MW of Power with Natural Gas
= (1,900,017 DT) . ($9.00/DT)

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= $ 17,100,156

e) What is the overall energy efficiency of the power station?

Solution/Answer:

Part (e) entails determination of the overall efficiency of the power generating station. Efficiency calculation, in this case study, requires knowledge of the total electrical energy (or power) produced and the total energy (or power) consumed through the combustion of natural gas. The output of the overall power generating system is ostensible from the given system output rating of 10 MW. The total heat consumption by the system is derived in parts (b), (c) and (d). The overall efficiency of the system would then be assessed by dividing the output power (or energy) by the input power (or energy).

From Part (c):

Total Energy Required Per Day, in BTU's = 5,205,526, 873 BTU's

Since there are 1.055kJ per BTU,
> **The Total Energy Input, Each Day, in kJ**
> > **= 5,205,526, 873 BTU's x 1.055 kJ/BTU**
> > **= 5,491,830,851 kJ**

This fuel energy usage can be converted into Watts of Mega Watts (MW) by dividing the energy usage by the total number of seconds in a day. This is because the **5,491,830,851 kJ** of energy is used over a period of a day.

∴ **The System Power Input**
> **= (5,491,830,851 kJ)/(24 hr x 3600 sec/hr)**
> **= 63,563 kJ/sec or 63,563 kW**

Since there are 1000 kW per MW,
> **The System Power Input, in MW**
> > **= (63,563 kW)/(1,000 kW/MW)**
> > **= 63.563 MW**

Since Power Output = 10.00 MW,   { Given }
Total Station Energy Efficiency, in Percent
$\qquad\qquad$ **= (Power Output / Power Input) x 100**
$\qquad\qquad$ **= 10 MW/ 63.563 MW x 100**
$\qquad\qquad$ **= 16%**

f) If heat is added to the steam turbines, would the steady flow energy process in the turbine system constitute an adiabatic process or a non-adiabatic process?

Solution/Answer:

The answer to Part (f) of the case study lies simply in the definition for adiabatic process.

Adiabatic process is a thermodynamic process in which no heat either enters or leaves the thermodynamic system boundary. In this case study, we assumed that heat is lost through the turbine casing. Therefore, this process is a **Non-Adiabatic Process.**

g) What is the change in entropy, Δs, in the turbine system?

Solution/Answer:

Part (g) involves computation of the change in entropy in the turbine segment of the overall thermodynamic process. As characteristic of the turbine stage of a typical heat engine cycle, there is a small change in entropy as the working fluid travels from the intake to the exit point of the turbine. This is evident as the entropy is assessed, through the superheated steam tables, for the incoming steam at 500°C and 2.5MPa, and the outgoing steam at 150°C and 50 kPa.

To determine the change in entropy, Δs, in the turbine system we need to identify s_1, the entropy of 500 °C, 2.5 MPa steam entering the steam turbine and s_2, the entropy of 150 °C, 50 kPa steam exiting the turbine.

$\quad$ **Δs = s_1 - s_2**

s_1 = **7.3234 kJ/kg. °K** {From superheated steam tables, See Table 2.4}

s_2 = **7.9401 kJ/kg. °K** {From superheated steam tables, See Table 2.4}

$$\therefore \Delta s = s_1 - s_2$$
$$= 7.3234 \text{ kJ/kg. °K} - 7.9401 \text{ kJ/kg. °K}$$
$$= -0.6167 \text{ kJ/kg. °K}$$

Note: The negative sign signifies the loss of heat through the turbine casing.

Case Study 2.1, US/Imperial Units, With Illustration of Interpolation Method:

Technical feasibility of a toping cycle cogenerating power plant is being studied at Station "Zebra." This facility is to be stationed in a remote Arctic region. The objective of this plant is to produce steam and generate electricity for an Arctic Environmental Monitoring and Deep Sea Mining Facility. Due to saltwater corrosion risk, it has been established that local glacier ice will be harvested and utilized for steam production purposes. A natural gas boiler is to be used to generate steam. The average temperature of the glacier ice hovers at 14 °F, through the year. The glacier ice is to be melted and converted to 932 °F, 362.6 psia, steam; the steam enters the turbine at this temperature and pressure. The steam is discharged by the turbines at 302 °F and 7.252 psia. This discharged steam is used for the mining process and to heat the station. The condensate is used as potable, utility and process water. Assume that the turbine represents an open, steady flow, thermodynamic system. In other words, the SFEE, Steady Flow Energy Equation applies. Also assume the potable water demand and flow, along the thermodynamic process stages, to be relatively negligible. Station Zebra would operate on a 24/7 schedule. For simplicity, the thermodynamic process flow for this system is laid out in Figure 2.9. **Note:** In an actual project setting, developing such a process flow diagram would constitute the first order of business as an energy engineer begins analyzing this process.

a) Estimate the mass flow rate **ṁ** for generation of 10 MW electrical power if the rate of turbine casing heat loss, **Q̇**, is 28.435 BTU/s (or 0.03 MW), exit velocity of steam, **v₂**, is 114.83 ft/sec, entrance velocity of steam, **v₁**, is 49.21

ft/sec, steam exit elevation is $z_2 = 3.28$ft, steam intake elevation is $z_1 = 1.64$ ft. Assume electric power generator efficiency of 90%. Extrapolate the answer into approximate truck loads per hour. Assume truck capacity of approximately 353.15 ft^3.

b) Assume that the power station is generating 10 MW of electric power. Calculate the amount of total heat energy needed, in BTU's/hr, to convert 14 °F harvested ice to 932 °F, 362.6 psia steam per hour.

c) Calculate the volume, in cu-ft, of natural gas required to power up the station, each day. Assume 98% burner efficiency.

d) If the natural gas transportation cost is \$4.85/DT in addition to the well head or commodity cost stated in Table 2.5, what would be the annual fuel cost of operating this station?

e) What is the overall energy efficiency of the power station?

f) If heat is added to the steam turbines, would the steady flow energy process in the turbine system constitute an adiabatic process or a non-adiabatic process?

g) What is the change in entropy, Δs, in the turbine system?

Process Flow Diagram Case Study
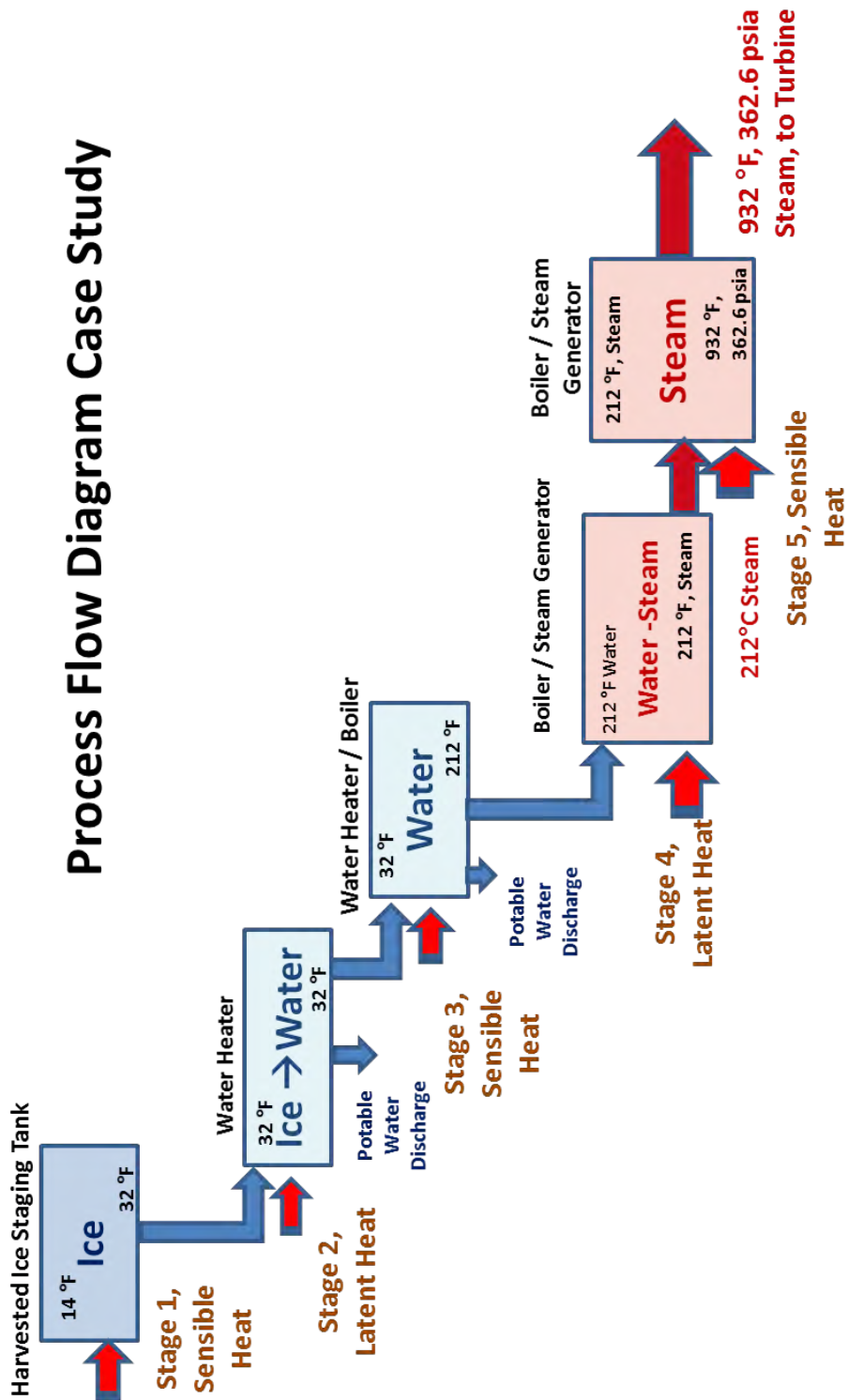
Figure 2.9, Thermodynamic Process Flow Diagram, Case Study 2.1

Energy Content and Cost Comparison

Fuel Type or Energy Source	Heat Content					Price Per Gallon*	Approx. Cost		US Sept. 2010 Average	
	DT/gal	BTU/gal	MMBTU/Barrel	BTU/Cu-ft.	DT/MCF		$/DT*	BTU/kWh	$/kWh	$/MMBTU
Propane	0.092	91,600	3.35	2,488		$1.42	$15.38			27.29
Diesel/No. 2 Fuel Oil	0.138	140,000	5.6			$2.65	$19.17			23.429
No. 6 Fuel Oil	0.144	143,888	6.8							
Natural Gas				1,034	1.034	$1.00	**$4.15**			4.15
Electricity							$30.01	3,412	0.1024	30.01

* Note: These costs represent a January 3, 2011 snapshot of wholesale or industrial market costs.

Table 2.5, Energy Content and Cost Comparison of Common Fuels

Case Study Solution Strategy

Before embarking on the analyses and solution for this case study, let's highlight some important facts from the case study statement.

1) As apparent from the case study statement, the working fluid or system consists of water, in various phases.

2) Water is introduced into the overall thermodynamic system in form of 14 °F ice and is then heated during various sensible and latent stages. It is, finally, introduced into the turbine as 932°F, 362.6 psia superheated steam.

3) Unlike a typical Rankin Cycle Heat Engine, the steam exhausted from the turbine is not condensed, pumped and recycled through the boilers to repeat the heat cycle. Instead, fresh ice is harvested, melted and introduced as working fluid.

4) Even though Figure 2.9 shows potable water being removed from the system at Stages 2 and 3, it is assumed to be negligible. Thus, supporting the assumption that mass flow rate of the working fluid stays constant through the system.

5) Fuel heat content and fuel cost information is available through Table 2.5.

6) Specific heat and latent heat data for water and ice are provided through Tables 3.6 and 3.7, for sensible heat and latent heat calculations, respectively.

7) Most of the data pertinent to the application of SFEE equation, at the turbine, is given, including the velocities, temperatures, pressures and elevations.

8) Final output of the power generating station is given in terms of the 10 MW rating of the generator and its stated efficiency of 90%.

The overall thermodynamic process flow can be tiered into stages that involve either sensible heating or latent heating. All heating stages of this comprehensive process are depicted in Figure 2.9. Each stage of the overall process is labeled with pertinent entry and exit temperature and pressure, as applicable. Each stage is named as either Sensible or Latent Stage. Furthermore, each stage shows the phase of water at point of entry and exit.

Approximate Specific Heat, c_p, for Selected Liquids and Solids			
Substance	c_p in J/gm K	c_p in cal/gm K or	Molar C_p
		BTU/lb F	J/mol K
Aluminum	0.9	0.215	24.3
Bismuth	0.123	0.0294	25.7
Copper	0.386	0.0923	24.5
Brass	0.38	0.092	...
Gold	0.126	0.0301	25.6
Lead	0.128	0.0305	26.4
Silver	0.233	0.0558	24.9
Tungsten	0.134	0.0321	24.8
Zinc	0.387	0.0925	25.2
Mercury	0.14	0.033	28.3
Alcohol(ethyl)	2.4	0.58	111
Water	4.186	1	75.2
Ice (-10 C)	2.05	0.49	36.9
Granite	0.79	0.19	...
Glass	0.84	0.2	

Table 2.6, Specific Heat, c_p, for Selected Liquids and Solids

	Latent Heat Fusion h_{sl}	Latent Heat Sublimation h_{ig}	Latent Heat Vaporization h_{fg}
kJ/kg	333.5	2838	2257
BTU/lbm	143.4	1220	970.3
kcal/kg	79.7	677.8	539.1

Table 2.7, Latent Heat for Phase Transformation of Water

a) Estimate the mass flow rate **ṁ** for generation of 10 MW electrical power output if the rate of turbine casing heat loss, **Q̇**, is 28.435 BTU/s (or 0.03 MW), exit velocity of steam, **v_2,** is 114.83 ft/sec, entrance velocity of steam, **v_1**, is 49.21 ft/sec, steam exit elevation is **z_2** = 3.28ft, steam intake elevation is **z_1** = 1.64 ft. Assume electric power generator efficiency of 90%. Extrapolate the answer into approximate truck loads per hour. Assume truck capacity of approximately 353.15 ft^3.

Solution:

This part of the case study can be analyzed and solved by simply focusing on the very last stage and applying the SFEE in form of Eq. 2.10. The computation of mass flow rate does not require assessment of the heat required at the various stages of the overall thermodynamic process, in this case study, because of the following key assumption included in the problem statement:

> "Also assume the potable water demand and flow, along the thermodynamic process stages to be relatively negligible. "

In other words the mass flow rate is assumed to be constant through out the process, and any discharge of water during individual stages of the overall process is negligible.

The turbine segment of the overall power generating system is illustrated in Figure 2.10, below. The enthalpy values are obtained from the superheated steam table excerpt in Table 2.8.

932°F, 362.6 psia Steam to Turbine.
Turbine Efficiency, η_t = 30%
h_i = 1489 BTU/lbm

302°F, 7.25 psia Low Pressure Steam for Space & Process Heat .
h_o = 1195.28 BTU/lbm

Generator Efficiency, η_g = 90% Electrical Output = 10 MW
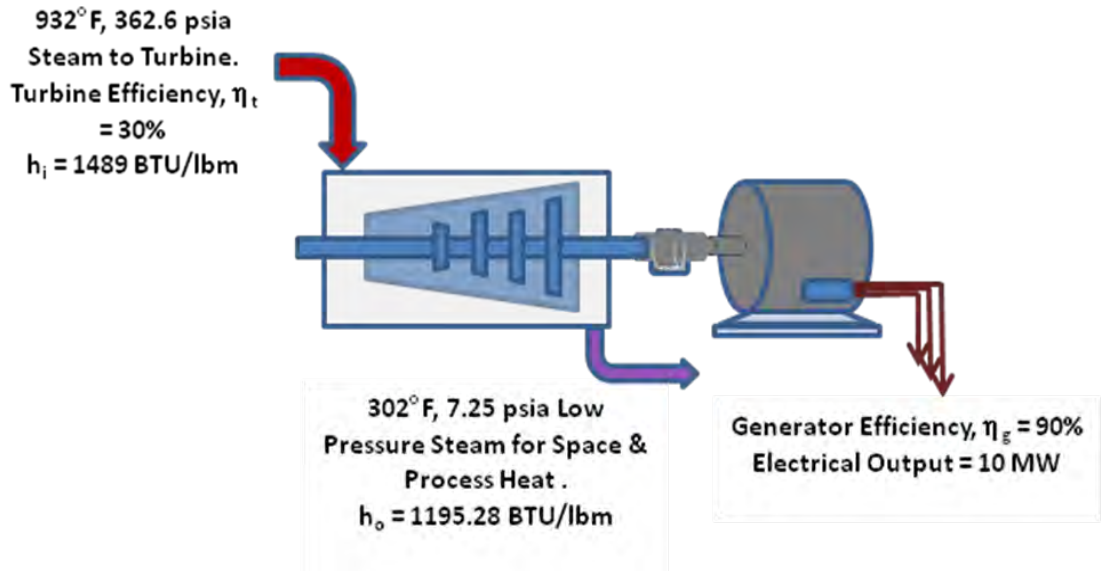
Figure 2.10, Case Study 2.1, Mass Flow Rate Analysis, US Units

Single and Double Interpolation of Steam Table Data for Enthalpy Determination:

Interpolation is often required when the retrieving data from tables such as the Saturated Steam Tables or the Superheated Steam Tables. Interpolation, specifically in steam tables, is needed when the given pressure, temperature or both don't coincide with the standard pressure and temperature values on the given tables.

This case study, in the US unit realm, offers an opportunity to study the interpolation method. Even though interpolation method is being illustrated on the basis of steam tables here, this technique can be employed for interpolation of other types of tabular data, as well. In this case study, enthalpy values need to be retrieved from the Superheated Steam Tables in Appendix B. The pressure and temperature for the initial (turbine entry) enthalpy, h_i, are 362.6 psia and 932°F, respectively. The pressure and temperature for the final (turbine exit) enthalpy, h_f, are 7.25 psia and 302°F, respectively. However, as you examine the superheated steam tables for these

parameters, exact match for this data is not available in the table. See Tables 2.8 and 2.9, below, for excerpts from the superheated steam tables in Appendix B.

As far as the initial point is concerned, h_i can be obtained through only one interpolation, or single interpolation; the interpolation associated with the temperature of 932°F, by rounding of the pressure to 360 psia. The magnitude of error in the enthalpy value, due to the 2.6 psia difference between 362.6 psia and 360 psia, is insignificant. This single interpolation approach was adopted in the derivation of the value of h_i = 1488.76 BTU/lbm.

The formula for single interpolation, applied between the stated or available enthalpy values for 900°F and 1000°F, at 360 psia, is as follows:

$$h_{i \text{ at } 932 \text{ °F, } 362.6 \text{ psia}} = h_{i \text{ at } 932 \text{ °F, } 360 \text{ psia}}$$
$$= ((h_{i \text{ at } 1000 \text{ °F, } 360 \text{ psia}} - h_{i \text{ at } 900 \text{ °F, } 360 \text{ psia}})/(1000°F - 900°F)).(932-900) + h_{i \text{ at } 900 \text{ °F, } 360 \text{ psia}}$$

Substituting enthalpy values and other given data from the superheated steam table excerpt, shown in Table 2.8 below:

$$h_{i \text{ at } 932 \text{ °F, } 362.6 \text{ psia}} = h_{i \text{ at } 932 \text{ °F, } 360 \text{ psia}}$$
$$= ((1525 \text{ BTU/lbm} - 1471.7 \text{ BTU/lbm})/(1000°F - 900°F)).(932°F - 900°F) + 1471.7 \text{ BTU/lbm}$$
$$= 1488.76 \text{ BTU/lbm}$$

Note: The available enthalpy values are circled in Table 2.8 below.

Properties of Superheated Steam US/Imperial Units					
Abs. **Press.** **psia**		**Temp.** **°F**	Note: v is in ft³/lbm, h is in BTU/lbm and s is in BTU/(lbm-°R)		
(Sat. Temp. °F)		500	700	900	1000
260 **(404.45)**	v	2.062	2.5818	3.0683	3.3065
	h	1262.5	1370.8	1475.2	1527.8
	s	1.5901	1.6928	1.7758	1.8132
360 **(434.43)**	v	1.446	1.8429	2.2028	2.3774
	h	1250.6	1365.2	1471.7	1525
	s	1.5446	1.6533	1.7381	1.7758
450 **(456.32)**	v	1.1232	1.4584	1.7526	1.8942
	h	1238.9	1360	1468.6	1522.4
	s	1.5103	1.6253	1.7117	1.7499
600 **(486.25)**	▱				
	v			1.3023	1.411
	h			1463.2	1518
	s			1.577	1.7159

Table 2.8, Superheated Steam Table Excerpt, US/Imperial Units

Double interpolation method is employed in deriving $\mathbf{h_f}$, the enthalpy value at 7.25 psia and 302°F. As apparent from the superheated steam tables in Appendix B, this value is not readily available and, therefore, double interpolation must be conducted between the enthalpy values given for 5 psi, 300°F, and 10 psi, 350°F, to derive $\mathbf{h_{f\ at\ 7.25\ psia\ and\ 302°F}}$. Where, $\mathbf{h_{f\ at\ 7.25\ psia\ and\ 302°F}}$ is the final enthalpy – enthalpy at the turbine exit point – at 7.25 psia and

302°F. The double interpolation approach, as applied here, will entail three steps.

First step involves determination of $h_{f, \text{ at 7.25 psia and 300°F}}$, the enthalpy value at 7.25 psia and 300°F. The enthalpy values available and used in this first interpolation step are circled in Table 2.9. The following formula sums up the mathematical approach to this first step:

$h_{f \text{ at 300 °F, 7.25 psia}}$

$= ((h_{f \text{ at 300 °F, 5 psia}} - h_{i \text{ at 300 °F, 10 psia}})/(10 \text{ psia} - 5 \text{ psia})).(10 \text{ psia} - 7.25 \text{ psia})$
$+ h_{f \text{ at 300 °F, 10 psia}}$

Substituting enthalpy values and other given data from superheated steam table excerpt, shown in Table 2.9 below:

$h_{f \text{ at 300 °F, 7.25 psia}}$

$= ((1194.8 \text{ BTU/lbm} - 1193.8)/(10 \text{ psia} - 5 \text{ psia})).(10 \text{ psia} - 7.25 \text{ psia}) + 1193.8 \text{ BTU/lbm}$

$= \textbf{1194.35 BTU/lbm}$

Second interpolation step involves determination of $h_{f, \text{ at 7.25 psia and 350°F}}$, the enthalpy value at 7.25 psia and 350°F. The enthalpy values available and used in this interpolation step are circled in Table 2.9. The following formula sums up the mathematical approach associated with this interpolation step:

$h_{f \text{ at 350 °F, 7.25 psia}}$

$= ((h_{f \text{ at 350 °F, 5 psia}} - h_{i \text{ at 350 °F, 10 psia}})/(10 \text{ psia} - 5 \text{ psia})).(10 \text{ psia} - 7.25 \text{ psia}) + h_{f \text{ at 350 °F, 10 psia}}$

Substituting enthalpy values and other given data from superheated steam table excerpt, shown in Table 2.9 below:

$h_{f \text{ at 350 °F, 7.25 psia}}$

$= ((\textbf{1218 BTU/lbm} - \textbf{1217.2})/(10 \text{ psia} - 5 \text{ psia})).(10 \text{ psia} - 7.25 \text{ psia}) + \textbf{1217.2 BTU/lbm}$

$= \textbf{1217.64 BTU/lbm}$

The final step in the double interpolation process, as applied in this case, involves interpolating between $h_{f \text{ at 300 °F, 7.25 psia}}$ and $h_{f \text{ at 350 °F, 7.25 psia}}$ the

enthalpy values derived in the first two steps above, to obtain the desired final enthalpy **h$_{f \text{ at } 302 \text{ °F, 7.25 psia}}$** .

The formula for this final step is as follows:

h$_{f \text{ at } 302\text{°F, 7.25 psia}}$

= ((**h$_{f \text{ at } 350 \text{ °F, 7.25 psia}}$** – **h$_{f \text{ at } 300 \text{ °F, 7.25 psia}}$**)/(**350**°F **- 300**°F))**.(302**°F **- 300**°F) +

h$_{f \text{ at } 300 \text{ °F, 7.25 psia}}$

Substituting enthalpy values derived in the first two steps above:

h$_{f \text{ at } 302\text{°F, 7.25 psia}}$

= ((1217.64 BTU/lbm – 1194.35 BTU/lbm)/(350°F - 300°F)).(302°F - 300°F) + 1194.35 BTU/lbm

h$_{f \text{ at } 302\text{°F, 7.25 psia}}$ = **1195.28 BTU/lbm**

Properties of Superheated Steam				
US/Imperial Units				
Abs. Press. psia (Sat. T, °F)	**Temp. °F**	Note: ν is in ft^3/lbm, h is in BTU/lbm and s is in BTU/(lbm-°R)		
		200	300	350	500
5 (162.18)	ν		90.248	96.254	
	h		1194.8	1218	
	s		1.937	1.9665	
10 (193.16)	ν	38.851	44.993	48.022	
	h	1146.4	1193.8	1217.2	
	s	1.7926	1.8595	1.8893	
15 (212.99)	ν		29.906		37.986
	h		1192.7		1287.3
	s		1.8137		1.9243
100 (327.82)	ν				5.5875
	h				1279.3
	s				1.7089

Table 2.9, Superheated Steam Table Excerpt, US/Imperial Units

With the key enthalpy values for turbine entry and exit points identified, we can proceed with the computation of the mass flow rate.

The **Open System Steady Flow Energy Equation** for power computation in the US or Imperial unit realm is:

$$\dot{Q} = \dot{m} \left[(h_2 - h_1) + 1/2 \cdot (v_2{}^2 - v_1{}^2)/(g_c \cdot J) + g \cdot (z_2 - z_1)/(g_c \cdot J) \right] + P_{shaft} \qquad \textbf{Eq. 2.12}$$

Rearrangement of Eq. 2.12 yields:

$$\dot{m} = (\dot{Q} - P_{shaft}) \, / \, [\, (h_2 - h_1) + 1/2 \cdot (v_2{}^2 - v_1{}^2)/(g_c.J)$$
$$+ \, g.(z_2 - z_1)/(g_c.J) \,] \qquad \textbf{Eq. 2.24}$$

Where,

h_1 = Enthalpy of the steam entering the turbine, in BTU/lbm.
h_2 = Enthalpy of the steam exiting the turbine, in BTU/lbm.
v_1 = Velocity of the steam entering the turbine, in ft/s.
v_2 = Velocity of the steam exiting the turbine, in ft/s.
z_1 = Elevation of the steam entering the turbine, in ft.
z_2 = Elevation of the steam exiting the turbine, in ft.
$\dot{Q}$ = Flow rate of heat added or removed from the turbine system, in BTUs/sec.
$\dot{m}$ = Mass flow rate of the system, in lbm/sec.
g = Acceleration due to gravity, 32.2 ft/s^2
g_c = Gravitational constant = 32.2 lbm-ft/lbf-sec^2
J = 778 ft-lbf/BTU

Given:
$\dot{Q}$ = - 0.03 MW = - 0.03x10^6 W = - 0.03x10^6 J/s

Since there are **1055 Joules per BTU**,

$$\dot{Q} = (- 0.03\text{x}10^6 \text{ J/s}) / (1055 \text{ J/BTU})$$
$$= \textbf{- 28.4 BTU/s}$$

$$P_{shaft} = (10 \text{ MW}) / \eta_g$$
$$= (10 \text{ MW}) / (0.9)$$
$$= 11.11\text{x}10^6 \text{ W}$$
$$= 11.11\text{x}10^6 \text{ J/s}$$

Since there are 1055 Joules per BTU,

$$P_{shaft} = (11.11\text{x}10^6 \text{ J/s}) / (1055 \text{ J/BTU})$$
$$= \textbf{10,530 BTU/s}$$

h₂ = 1195 BTU/lbm, from Appendix B and interpolation stated above.

h₁ = 1489 BTU/lbm from Appendix B and interpolation stated above.

v₂ = 114.83 ft/sec

v₁ = 49.21 ft/sec

z₂ = 3.28ft

z₁ = 1.64 ft

By applying Eq. 2.24:

$$\dot{m} = (\dot{Q} - P_{shaft}) / [(h_2 - h_1) + 1/2 \cdot (v_2^2 - v_1^2)/(g_c \cdot J) + g \cdot (z_2 - z_1)/(g_c \cdot J)] \qquad \text{Eq. 2.24}$$

ṁ = (- 28.4 **BTU/s - 10,530 BTU/s) / [(1195 BTU/lbm – 1489 BTU/lbm) + 1/2 . ((114.83 ft/sec)² – (49.21 ft/sec)²)/(32.2 lbm-ft/lbf-sec². 778 ft-lbf/BTU) + (32.2 ft/s²).(3.28ft – 1.64 ft)/(32.2 lbm-ft/lbf-sec² .778 ft-lbf/BTU)]**

For clarity, the **ṁ** computation equation, Eq. 2.24, with known values substituted, can be stated alternatively as:

$$\dot{m} = \cfrac{(-28.4\,BTU/s - 10{,}530\,BTU/s)}{\left[\left(1195\,BTU/lbm - 1489\,BTU/lbm\right) + 1/2\,\cfrac{(114.83\,ft/s)^2 - (49.21\,ft/s)^2}{(32.2\,lbm-ft/lbf-s^2).(778\,ft-lbf/BTU)} + (32.2\,ft/s^2).\cfrac{(3.28\,ft - 1.64\,ft)}{(32.2\,lbm-ft/lbf-s^2).(778\,ft-lbf/BTU)}\right]}$$

ṁ = 35.94 lbm/sec ,

Or,

ṁ = (35.94lbm/sec) . (3600 sec/hour)

ṁ = 129,382 lbm/hr or 58,687 kg/hr

Or,

ṁ = {(35.94lbm/2000 lbm/ton)/sec }.(3600 sec/hour)

ṁ = 65 tons/hour

The density data for of ice and water is as follows:

Density of Water = 1000 kg/m³ or **62.4 lb/cu.-ft at 4 °C**.

Density of ice = 917 kg/m³ or **57.26 lbm/cu-ft**

Since our objective is to determine the volumetric flow rate of ice, we will introduce the density of ice in determination of the volumetric flow rate.

Therefore, the volumetric flow rate, $\dot{V}$, would be:

$$\dot{V} = (35.94 \text{ lbm/sec}).(3600 \text{ sec/hr})/(57.26 \text{ lbm/ft}^3)$$

$$\dot{V} = 2260 \text{ cu-ft/hr}$$

At an estimated **353.15 ft³ per truck load**, this volumetric mass flow rate would amount to:

$$\dot{V} = (2260 \text{ cu-ft/hr})/ 353.15 \text{ ft}^3 \text{ per truck load}$$

Or,

$$\dot{V} = 6.4 \text{ truck loads per hour}$$

b) Assume that the power station is generating 10 MW of electric power. Calculate the amount of heat needed, in BTU's/hr, to convert 14 °F harvested ice to 932 °F, 362.6 psia steam, per hour.

Solution:

Similar to the metric unit version, part (b) of the US unit version of Case Study 2.1 requires accounting for heat added during each of the five (5) stages of the overall process. Therefore, this part is divided into five sub-parts, each involving either sensible or latent heat calculation, based on the entry and exit temperature and phase status.

Table 2.6 lists specific heat for water and ice. These heat values will be used in the sensible heat calculations associated with Part (b). Table 2.7 lists latent heat values for water. These values will be used to compute the latent heats associated with stages that involve phase transformation.

(i) Calculate the heat required to raise the temperature of the ice from 14 °F to 32 °F.

Since there is no change in phase involved, the entire heat absorbed by the ice (working substance) in this stage would be sensible heat.

First stage of the overall power generating system is illustrated in Figure 2.11, below.
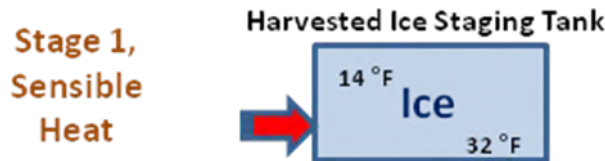
Figure 2.11, Case Study 2.1, US Units, Stage 1 Sensible Heat Calculation

Given:

T_i = 14 °F
T_f = 32 °F
c_{ice} = 0.49 BTU/lbm.°F
{Table 2.6}

Utilizing the given information:

$\Delta T = T_f - T_i$
$\therefore \Delta T = 32 - 14$ °F
$= 18$ °F

Since ΔT represents the *change* in temperature and not a specific absolute temperature,

$\Delta T = 10$ °F $= 10$°R

The mathematical relationship between sensible heat, mass of the working substance, specific heat of the working substance and change in temperature can be stated as:

$Q_{s(heat\ ice)} = m \cdot c_{ice} \cdot \Delta T$
Eq. 2.14

And,

$\dot{Q}_{s(heat\ ice)} = \dot{m} \cdot c_{ice} \cdot \Delta T$ **Eq. 2.15**

Where,

$Q_{s(heat\ ice)}$ = Sensible heat required to heat the ice over ΔT

$\dot{Q}_{s(heat\ ice)}$ = Sensible heat flow rate required to heat the ice over ΔT

m = Mass of ice being heated

c$_{ice}$ = Specific heat of ice = **0.49 BTU/lbm.°F= 0.49 BTU/lbm.°R**

Since **$\Delta°F = \Delta°R$**

ΔT = Change in temperature, in **°F or °R = 18°F= 18°R**

ṁ = Mass flow rate of water/ice = **35.94 lbm/sec**, or, **129,382 lbm/hr**
as computed in part (a)

Then, by applying **Eq. 2.15**:

$\dot{Q}_{s(heat\ ice)}$ = **(129,382 lbm/hr) . (0.49 BTU/lbm.°F) . (18°F)**

Or,

$\dot{Q}_{s(heat\ ice)}$ = **1,141,149 BTU/hr**

(ii) Calculate the heat required to melt the ice at 32 °F.

Since change in phase *is* involved in this case, the heat absorbed by the ice (working substance) in this stage would be latent heat.

The 2nd stage of the overall power generating system is illustrated in Figure 2.12, below.
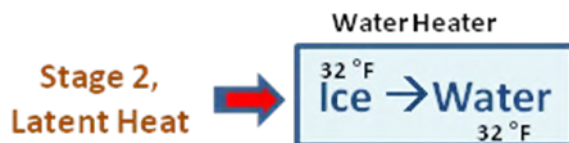
Figure 2.12, Case Study 2.1, US Units, Stage 2 Latent Heat Calculation

The mathematical relationship between latent heat, mass of the working substance, and the heat of fusion of ice can be stated as:

$Q_{l(latent\ ice)} = h_{sl\ (ice)}$ **. m**   **Eq. 2.16**

And,

$\dot{Q}_{l(latent\ ice)} = h_{sl\ (ice)}$ **. ṁ**   **Eq. 2.17**

Where,

$Q_{l(latent\ ice)}$ = Latent heat required to melt a specific mass of ice, isothermally

$\dot{Q}_{l(latent\ ice)}$ = Latent heat flow rate required to melt a specific mass of ice, isothermally, over a period of time

m = Mass of ice being melted

ṁ = Mass flow rate of water/ice = **129,382 lbm/hr** as computed in part (a)

$h_{sl\ (ice)}$ = Heat of fusion for Ice = **143.4 BTU/lbm** {Table 2.7}

Then, by application of **Eq. 2.17**:

$$\dot{Q}_{l(latent\ ice)} = h_{sl\ (ice)} \cdot \dot{m}$$

$$\dot{Q}_{l(latent\ ice)} = (143.4\ BTU/lbm) \cdot (129,382\ lbm/hr)$$

$$\dot{Q}_{l(latent\ ice)} = \textbf{18,553,380 BTU/hr}$$

Note that the specific heat required to melt ice is called heat of fusion because of the fact that the water molecules come closer together as heat is added in the melting process. The water molecules are held apart at specific distances in the crystallographic structure of solid ice. The heat of fusion allows the molecules to overcome the crystallographic forces and **"fuse"** to form liquid water. This also explains why the density of water is higher than the density of ice.

(iii) Calculate the heat reqd. to heat the water from 32 °F to 212 °F:

The 3rd stage of the overall power generating system is illustrated in Figure 2.13, below. Since no phase change is involved in this stage, the heat absorbed by the water in this stage would be sensible heat.
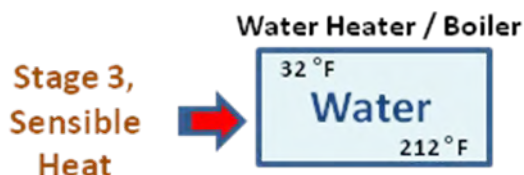
Figure 2.13, Case Study 2.1, US Units, Stage 3 Sensible Heat Calculation

Given:

T_i = **32 °F**

T_f = **212 °F**

c_{ice} = Specific heat of ice = **1.0 BTU/lbm.°F**

     = **1.0 BTU/lbm.°R** {Since $\Delta °F = \Delta °R$}   {Table 2.6}

Utilizing the given information:

$\Delta T = T_f - T_i$

$\therefore \Delta T = 212 °F – 32 °F$

       = 180 °F

Since **ΔT** represents the *change* in temperature and not a specific absolute temperature,

$\therefore \Delta T = 180 °R = 180 °F$

The mathematical relationship between sensible heat, mass of the working substance, specific heat of water (the working substance), and change in temperature can be stated as:

$Q_{s(water)} = m \cdot c_{p\text{-}water} \cdot \Delta T$ **Eq. 2.18**

And,

$\dot{Q}_{s(water)} = \dot{m} \cdot c_{p\text{-}water} \cdot \Delta T$ **Eq. 2.19**

Where,

$Q_{s(water)}$ = Sensible heat required to heat the water over ΔT

$\dot{Q}_{s(water)}$ = Sensible heat *flow rate* required to heat the water over ΔT

m = Mass of water being heated

$c_{p\text{-}water}$ = Specific heat of water = **1 BTU/lbm. °R**

     = **1 BTU/lbm.°F**   {Table 7 – 6}

$\dot{m}$ = Mass flow rate of water = **129,382 lbm/hr** as computed in part (a)

**ΔT** = Change in temperature, in **°F or °R = 180 °F**

Then, by applying **Eq. 2.19**:

$\dot{Q}_{s(water)} = \dot{m} \cdot c_{p\text{-}water} \cdot \Delta T$

$$\dot{Q}_{s(water)} = (129{,}382 \text{ lbm/hr}) . (1 \text{ BTU/lbm.°F}) . (180 \text{ °F})$$

$$\dot{Q}_{s(water)} = \textbf{23,288,760 BTUs/hr}$$

(iv) Calculate the heat required to convert 212 °F water to 212 °F steam:

The 4th stage of the overall power generating system is illustrated in Figure 2.14, below. Since change in phase *is* involved in this case, the heat absorbed by the water in this stage would be latent heat.

Boiler / Steam Generator
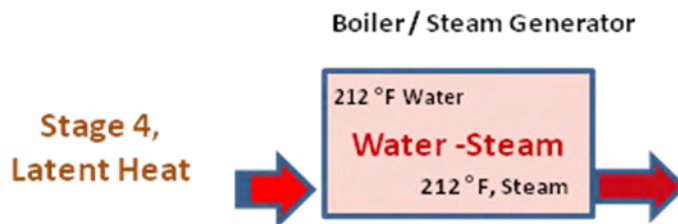
Figure 2.14, Case Study 2.1 Stage 4 Latent Heat Calculation

The mathematical relationship between latent heat of vaporization for water, $h_{fg(water)}$, mass of the water, and the total heat of vaporization of water, $Q_{l(latent\ water)}$, can be stated as:

$$Q_{l(latent\ water)} = h_{fg\ (water)} . m \qquad\qquad \textbf{Eq. 2.20}$$

And,

$$\dot{Q}_{l(latent\ water)} = h_{fg\ (water)} . \dot{m} \qquad\qquad \textbf{Eq. 2.21}$$

Where,

$Q_{l(latent\ water)}$ = Latent heat of vaporization of water required to evaporate a specific mass of water, isothermally

$\dot{Q}_{l(latent\ water)}$ = Latent heat of vaporization *flow rate* required to evaporate a specific mass of water, isothermally, over a given period of time

m = Mass of water being evaporated

$\dot{m}$ = Mass flow rate of water = **129,382 lbm/hr,** same as part (a) (i)

$h_{fg\ (water)}$ = latent heat of vaporization for water

= **970.3 BTU/lbm** {From the steam tables and Table 2.7}

Then, by application of **Eq. 2.21**:

$$\dot{Q}_{l(latent\ water)} = h_{fg\ (water)}\ .\ \dot{m}$$

$$\dot{Q}_{l(latent\ water)} = (970.3\ BTU/lbm).\ (129,382\ lbm/hr)$$

$$\dot{Q}_{l(latent\ water)} = 125,539,355\ BTUs/hr$$

(v) Calculate the heat reqd. to heat the steam from 212°F, 1-atm (102 KPa, or 1-bar) to 932 °F, 362.6 psia superheated steam

The 5th stage of the overall power generating system is illustrated in Figure 2.15, below. Since this stage involves no phase change, the heat absorbed by the steam is sensible heat.

In superheated steam phase, the heat required to raise the temperature and pressure of the steam can be determined using the enthalpy difference between the initial and final conditions.

Boiler / Steam Generator
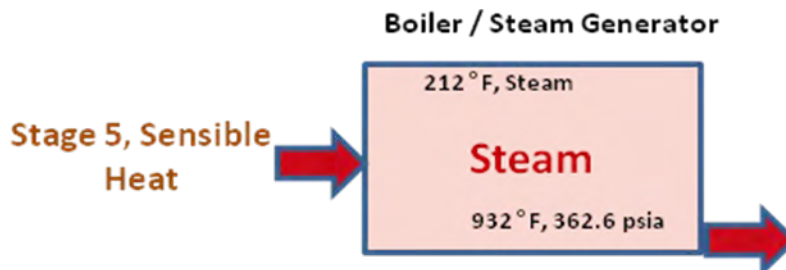
Figure 2.15, Case Study 2.1 Stage 5 Sensible Heat Calculation

Given:

$T_i = 212\ °F$

$P_i = 1\text{-}Atm.$ Note: At 212 °F, the saturation pressure is 1- Atm, 1-Bar, or 102 kPa

$T_f = 932\ °F$

$P_f = 362.6\ psia$

For the initial and final temperature and pressure conditions stated above, the enthalpy values - as read from superheated steam table excerpt in Table 2.8, as interpolated in part (a) of this case study, and from Table 2.10 - are as follows:

$$h_{i \text{ at } 212°F, \text{ 1-Atm}} = \textbf{1149.4 BTU/lbm}$$

$$h_{f \text{ at } 932 °F, \text{ 362.6 psia}} = h_{f \text{ at } 932 °F, \text{ 360 psia}} = \textbf{1488.76 BTU/lbm}$$

Equations for determining the heat required to boost the steam from **212°F, 1-Atm** to **932 °F, 362.6 psia** are as follows:

$$\Delta Q_{steam} = (h_f - h_i) \cdot m \qquad\qquad \textbf{Eq. 2.22}$$

$$\dot{Q}_{steam} = (h_f - h_i) \cdot \dot{m} \qquad\qquad \textbf{Eq. 2.23}$$

Where,

ΔQ_{steam} = Addition of heat required for a specific change in enthalpy
$\dot{Q}_{steam}$ = Rate of addition of heat for a specific change in enthalpy
h_i = Initial enthalpy
h_f = Final enthalpy
m = Mass of steam being heated
$\dot{m}$ = **129,382 lbm/hr,** same as part (a)

Then, by applying **Eq. 2.23**:

$$\dot{Q}_{steam} = (h_f - h_i) \cdot \dot{m}$$

$$\dot{Q}_{steam} = (1488.76 \text{ BTU/lbm} - 1149.4 \text{ BTU/lbm}) \cdot (129,382 \text{ lbm/hr})$$

$$\dot{Q}_{steam} = \textbf{43,907,076 BTU/hr}$$

Properties of Saturated Steam By Pressure

US/Imperial Units

Abs. Press. psia	Temp. °F	Specific Volume ft³/lbm		Enthalpy Btu/lbm			Entropy Btu/(lbm.°R)		Abs. Press. psia
		Sat. Liquid v_L	Sat. Vapor v_V	Sat. Liquid h_L	Evap. h_{fg}	Sat. Vapor h_V	Sat. Liquid s_L	Sat. Vapor s_V	
1.0	101.69	0.016137	333.51	69.728	1036	1105.4	0.1326	1.9776	1.0
4.0	152.91	0.016356	90.628	120.89	1006.4	1126.9	0.2198	1.8621	4.0
14.0	209.52	0.016697	28.048	177.68	972.0	1149.4	0.3084	1.7605	14.0
100	327.82	0.017736	4.4324	298.57	889.2	1187.5	0.4744	1.6032	100

Table 2.10, Saturated Steam Table Excerpt, US/Imperial Units

After assessing the heat added, per hour, during each of the five (5) stages of the steam generation process, add all of the heat addition rates to compile the total heat addition rate for the power generating station. The tallying of total heat is performed in BTU's/hr as well as kJ/hr.

Total Heat Addition Rate in BTU's/hr:

Total Heat Required to Generate 932°F, 362.6 psia steam from 1°F Ice, at 129,382 lbm/hr
> = 1,141,149 BTU/hr + 18,553,380 BTU/hr + 23,288,760 BTUs/hr + 125,539,355 BTUs/hr + 43,907,076 BTU/hr
> = **212,429,719 BTU/hr**

c) Calculate the volume, in cu-ft, of natural gas required to power up the station, each day. Assume 98% burner efficiency.

Solution:

This part of Case Study 2.1 requires computation of the amount (volume) of natural gas required to power up the station each day. This calculation is straight forward after the derivation of the total energy required, per hour, in part (b). However, the hourly energy requirement must be "scaled up" to account for the 98% efficiency of the boiler burner.

The hourly energy requirement, in kJ or BTU's, can be extended into daily usage. The daily energy usage can then be converted into the volume of natural gas required, based on natural gas energy content listed in Table 2.5.

Total Energy Required Per Day = (212,429,719 BTU/hr)/(0.98) x 24 hr
> = **5,098,313,255 BTUs**

Since Natural Gas Energy Content = **1034 BTU/cu-ft,**

Total volume of natural gas required per day
> = (**5,098,313,255 BTUs**) / **1034 BTU/cu-ft**
> = **4,930,670 cu-ft**

d) If the natural gas transportation cost is $ 4.85/DT in addition to the well head or commodity cost stated in Table 2.5, what would be the annual fuel cost of operating this station?

Solution:

Part (d) of the case study relates to the computation of total annual cost of fuel for the power generation station. This requires the extrapolation of daily energy consumption into *annual* energy consumption. The annual energy consumption is then multiplied with the *total*, delivered, cost rate in $/DT to obtain the annual cost in dollars.

From part (b), the heat energy required for operating the power station = 196,909,695 BTU/hr

Then, based on 24/7 operating schedule assumption, the total energy required per year would be:

  = (212,429,719 BTU/hr)/(0.98) x 8760 hrs/year
  = 1.8989 E+12 BTU's

Since there are 1,000,000 BTU's per DT, the Total Annual Energy Consumption in DT would be:

  = **1.8989 E+12 BTU's / (1,000,000 BTU's/DT)**
  = **1,898,862 DT's**

The total cost of natural gas, per DT =
   Cost at the Source/Well + Transportation Cost
     = $4.15/DT + $4.85/DT
     = $9.00/DT

Then, the Total Annual Cost for Producing 10MW of Power with Natural Gas
  = $ 17,089,754

e) What is the overall energy efficiency of the power station ?

Solution/Answer:

Part (e) entails determination of the overall efficiency of the power generating station. Efficiency calculation, in this case study, requires knowledge of the total electrical energy (or power) produced and the total energy (or power) consumed through the combustion of natural gas. The output of the overall power generating system is ostensible from the given system output rating of 10 MW. The total heat consumption by the system is derived in parts (b), (c) and (d). The overall efficiency of the system would then be assessed by dividing the output power (or energy) by the input power (or energy).

From Part (c):

Total Energy Required Per Day, in BTU's = 5,098,313,255 BTUs

Since there are 1.055kJ per BTU,

> **The Total Energy Input, Each Day, in kJ**
> = 5,098,313,255 BTUs x 1.055 kJ/BTU
> **= 5,378,720,484 kJ**

This fuel energy usage can be converted into Watts of Mega Watts (MW) by dividing the energy usage by the total number of seconds in a day. This is because the **5,378,720,484 kJ** of energy is used over a period of a day.

∴ **The System Power Input**
> = (5,378,720,484 kJ)/(24 hr x 3600 sec/hr)
> **= 62,254 kJ/sec or   62,254 kW**

Since there are 1000 kW per MW,
> **The System Power Input, in MW**
> = (62,254 kW)/(1,000 kW/MW)
> **= 62.254 MW**

Since Power Output = 10.00 MW   { Given }

Total Station Energy Efficiency, in Percent
> **= Power Output / Power Input x 100**

$$= 10 \text{ MW}/ 62.254 \text{MW} \times 100$$
$$= \textbf{16\%}$$

f) If heat is added to the steam turbines, would the steady flow energy process in the turbine system constitute an adiabatic process or a non-adiabatic process?

Solution/Answer:

The answer to Part (f) of the case study lies simply in the definition for adiabatic process.

Adiabatic process is a thermodynamic process in which no heat either enters or leaves the thermodynamic system boundary. In this case study, we assumed that heat is lost through the turbine casing. Therefore, this process is a **Non-Adiabatic Process.**

g) What is the change in entropy, Δs, in the turbine system?

Solution/Answer:

Part (g) involves computation of the change in entropy in the turbine segment of the overall thermodynamic process. As characteristic of the turbine stage of a typical heat engine cycle, there is a small change in entropy as the working fluid travels from the intake to the exit point of the turbine. This is evident as the entropy is assessed, through the superheated steam tables, for the incoming steam at 932 °F and 362.6 psia, and the outgoing steam at 302°Fand 7.252 psia.

To determine the change in entropy, Δs, in the turbine system we need to identify s_1, the entropy of 932 °F, 362.6 psia steam entering the steam turbine and s_2, the entropy of 302 °F, 7.252 psia steam exiting the turbine. As evident from superheated steam table excerpts in Tables 3.8 and 7.9, the desired entropy values are not readily available. Interpolation is required to obtain the entropy values at the entry and exit points of the turbine.

The formula for single interpolation, applied between the stated or available entropy values for 900°F and 1000°F, at 360 psia, is as follows:

Si at 932 °F, 362.6 psia $=$ **S**i at 932 °F, 360 psia

$= ((\textbf{S}_{\text{i at 1000 °F, 360 psia}} - \textbf{S}_{\text{i at 900 °F, 360 psia}})/(1000°\text{F} - 900°\text{F})).(932-900)$

$+ \textbf{S}_{\text{i at 900 °F, 360 psia}}$

Substituting entropy values and other given data from superheated steam table excerpt, shown in Table 2.11 below:

Si at 932 °F, 362.6 psia $=$ **S**i at 932 °F, 360 psia

$= ((1.7758 \textbf{ BTU/lbm-}°\text{F} - 1.7381 \textbf{ BTU/lbm-}°\text{F})/(1000°\text{F} - 900°\text{F})).(932°\text{F} - 900°\text{F}) + 1.7381 \textbf{ BTU/lbm-}°\text{F}$

$\therefore$ **S**i at 932 °F, 362.6 psia $= 1.7502 \textbf{ BTU/lbm-}°\text{F}$

Note: The available entropy values are circled in Table 2.11 below.

Properties of Superheated Steam US/Imperial Units					
Abs. Press. psia (Sat. T, °F)	**Temp. °F**	Note: v is in ft^3/lbm, h is in BTU/lbm and ***s*** is in BTU/(lbm-°R)			
		500	700	900	1000
260 **(404.45)**	v	2.062	2.5818	3.0683	3.3065
	h	1262.5	1370.8	1475.2	1527.8
	s	1.5901	1.6928	1.7758	1.8132
360 **(434.43)**	v	1.446	1.8429	2.2028	2.3774
	h	1250.6	1365.2	1471.7	1525
	s	1.5446	1.6533	1.7381	1.7758
450 **(456.32)**	v	1.1232	1.4584	1.7526	1.8942
	h	1238.9	1360	1468.6	1522.4
	s	1.5103	1.6253	1.7117	1.7499
600 **(486.25)**	v			1.3023	1.411
	h			1463.2	1518
	s			1.577	1.7159

Table 2.11, Superheated Steam Table Excerpt, US/Imperial Units

Double interpolation method is employed in deriving s_f, the enthalpy value at 7.25 psia and 302°F. As apparent from the superheated steam tables in Appendix B, this value is not readily available and, therefore, double interpolation must be conducted between the enthalpy values given for 5 psi, 300°F, and 10 psi, 350°F, to derive $s_{f\ at\ 7.25\ psia\ and\ 302°F}$. Where, $s_{f\ at\ 7.25\ psia\ and\ 302°F}$ is the final enthalpy – enthalpy at the turbine exit point – at 7.25 psia and

302°F. The double interpolation approach, as applied here, will entail three steps.

First step involves determination of $s_{f, \text{ at } 7.25 \text{ psia and } 300°F}$, the enthalpy value at 7.25 psia and 300°F. The enthalpy values available and used in this first interpolation step are circled in Table 2.9. The following formula sums up the mathematical approach to this first step:

s_f at 300 °F, 7.25 psia

 = ((s_f at 300 °F, 5 psia - s_i at 300 °F, 10 psia)/(10 psia -5 psia)).(10 psia-7.25 psia) + s_f at 300 °F, 10 psia

Substituting enthalpy values and other given data from superheated steam table excerpt, shown in Table 2.9 below:

s_f at 300 °F, 7.25 psia

 = ((**1.937 BTU/lbm-°F – 1.8595 BTU/lbm-°F**)/(**10 psia -5 psia**)).(**10 psia-7.25 psia**) + **1.8595 BTU/lbm-°F**

         = **1.902125 BTU/lbm-°F**

Second interpolation step involves determination of $s_{f, \text{ at } 7.25 \text{ psia and } 350°F}$, the enthalpy value at 7.25 psia and 350°F. The enthalpy values available and used in this interpolation step are circled in Table 2.12. The following formula sums up the mathematical approach associated with this interpolation step:

s_f at 350 °F, 7.25 psia

 = ((s_f at 350 °F, 5 psia - s_i at 350 °F, 10 psia)/(10 psia -5 psia)).(10 psia-7.25 psia) + s_f at 350 °F, 10 psia

Substituting enthalpy values and other given data from superheated steam table excerpt, shown in Table 2.12 below:

s_f at 350 °F, 7.25 psia

 = ((**1.9665 BTU/lbm-°F – 1.8893 BTU/lbm-°F**)/(**10 psia -5 psia**)).(**10 psia-7.25 psia**) + **1.8893 BTU/lbm-°F**

         = **1.93176 BTU/lbm-°F**

The final step in the double interpolation process, as applied in this case, involves interpolating between $s_{f \text{ at } 300 °F, 7.25 \text{ psia}}$ and $s_{f \text{ at } 350 °F, 7.25 \text{ psia}}$ the

enthalpy values derived in the first two steps above, to obtain the desired final enthalpy **S**f **at 302 °F, 7.25 psia** .

The formula for this final step is as follows:

Sf **at 302°F, 7.25 psia**

 = ((**S**f **at 350 °F, 7.25 psia** – **S**f **at 300 °F, 7.25 psia**)/($350°\text{F} - 300°\text{F}$)).($302°\text{F} - 300°\text{F}$) + **S**f **at 300 °F, 7.25 psia**

Substituting enthalpy values derived in the first two steps above:

Sf **at 302°F, 7.25 psia**

 = ((**1.932 BTU/lbm-**°F **– 1.9021 BTU/lbm-**°F)/($350°\text{F} - 300°\text{F}$)).($302°\text{F} - 300°\text{F}$) + **1.902125 BTU/lbm-**°F

Sf **at 302°F, 7.25 psia** = **1.9033BTU/lbm-**°F

Properties of Superheated Steam					
US/Imperial Units					
Abs. Press. psia (Sat. T, °F)		**Temp. °F**	Note: v is in ft^3/lbm, h is in BTU/lbm and s is in BTU/(lbm-°R)		
		200	300	350	500
5 (162.18)	v		90.248	96.254	
	h		1194.8	1218	
	s		1.937	1.9665	
10 (193.16)	v	38.851	44.993	48.022	
	h	1146.4	1193.8	1217.2	
	s	1.7926	1.8595	1.8893	
15 (212.99)	v		29.906		37.986
	h		1192.7		1287.3
	s		1.8137		1.9243
100 (327.82)	v				5.5875
	h				1279.3
	s				1.7089

Table 2.12, Superheated Steam Table Excerpt, US/Imperial Units

With the key enthalpy values for turbine entry and exit points identified, we can proceed with the computation of **Δs:**

s_f at 302°F, 7.25 psia $= \textbf{1.9033BTU/lbm-}°F$

s_i at 932 °F, 362.6 psia $= \textbf{1.7502 BTU/lbm-}°F$

Then,

$$\Delta s = s_{f \text{ at } 302°F, 7.25 \text{ psia}} - s_{i \text{ at } 932 °F, 362.6 \text{ psia}}$$

$$= 1.7502 \text{ BTU/lbm-}°F - 1.9033 \text{BTU/lbm-}°F$$

$$= - 0.1531 \text{ BTU/lbm-}°F$$

Note: The negative sign signifies the loss of heat through the turbine casing.

Self-assessment Problems & Questions – Segment 2

1**.** Why is the efficiency of this power plant in Case Study 2.1 rather low (17%)**?**

2. Using the steam tables in Appendix B and the Double Interpolation Method described in Case Study 2.1, US Unit Version, determine the exact enthalpy of a superheated steam at a pressure of 400 psia and temperature of 950°F.

3. In Case Study 2.1, as an energy engineer you have been retained by Station Zebra to explore or develop an alternative integrated steam turbine and electric power generating system that is capable of generating 10 MW of power with only 60 truck loads, or 54,432 kg, of ice per hour. With all other parameters the same as in the original Case Study 2.1 scenario, determine the total heat flow rate needed, in kJ/hr, to produce 10 MW of electrical power.

4. If all of the working fluid, or steam, discharged from the turbine in Case Study 2.1 is reclaimed, reheated and returned to the turbine, what would be the overall system efficiency?

Segment 3

Thermodynamic Processes

Topics:

- Thermodynamic processes
- Heat engine cycles
- Steam turbines
- Temperature-enthalpy diagrams
- Pressure-enthalpy diagrams
- Pressure-volume diagrams
- Temperature-entropy diagrams
- Practical examples and associated case study.

Introduction:

This segment explores some of the mainstream thermodynamic processes, heat engines and heat engine cycles. Fundamentals of thermodynamic processes, heat engines, heat engine cycles and associated systems are explained and illustrated through process flow diagrams, graphs, tables and pictures. Practical significance, application, analytical methods and computational techniques associated with heat engine cycles and thermodynamic processes are demonstrated through case study, examples and self-assessment problems.

Thermodynamic Processes

Thermodynamic processes are processes that entail heat, internal energy, enthalpy, entropy, work, pressure, temperature and volume. In this section, we will explore the following thermodynamic processes and illustrate these processes with practical examples:

1. Adiabatic Process
2. Isenthalpic Process
3. Isochoric Process
4. Isothermal Process
5. Isobaric Process
6. Isentropic Process

Adiabatic Process:

Adiabatic process is a thermodynamic process in which no heat either enters or leaves the thermodynamic system boundary. An adiabatic process can also be explained through the following mathematical statements or equations:

$$\Delta U \ = \ - W \qquad\qquad\qquad \textbf{Eq. 3.1}$$

$$\Delta Q \ = \ 0 \qquad\qquad\qquad \textbf{Eq. 3.2}$$

Equations 3.1 and 3.2 essentially state that in an adiabatic process, wherein no heat is gained or lost, any work performed on the system or by the system is transformed into a net change in the internal energy of the system. As specifically stated above, Eq. 3.1 represents a scenario where negative work is involved. In other words, work is being performed by the surroundings onto the system. And, since no heat is transferred to or from the environment in an adiabatic process, the work performed by the surroundings onto the system, in this case, is converted into an equivalent amount of increase in the internal energy of the system. This explanation of the adiabatic process is validated by the fact the units for work "W" and internal energy "U" are the same; Joules in the SI/Metric realm and BTU's in the US/Imperial realm.

Equation 3.1 can be restated as:

$$- \Delta U = \ \ W \qquad\qquad\qquad \textbf{Eq. 3.1a}$$

It is important to note that while this restatement of the Eq. 3.1 keeps the equation mathematically equivalent to the original version, the physical significance changes. Equation 3.1a represents a scenario where work is positive and is performed by the system onto the environment or surroundings. Since this is an adiabatic process, there is no transfer or exchange of heat. Therefore, in this case, work is performed by the system, onto the surroundings, at the expense of the internal energy of the system. The negative ΔU signifies a reduction in the internal energy of the system.

Work performed in adiabatic processes, such as that performed by the compressor on the refrigerant, would be represented by Eq. 3.3.

$$W = \int_{V_1}^{V_2} p.dV$$

Eq. 3.3

In a reversible adiabatic process, such as the compression stroke in an internal combustion gasoline engine, the product of pressure and volume is represented as shown in **Eq. 3.4.**
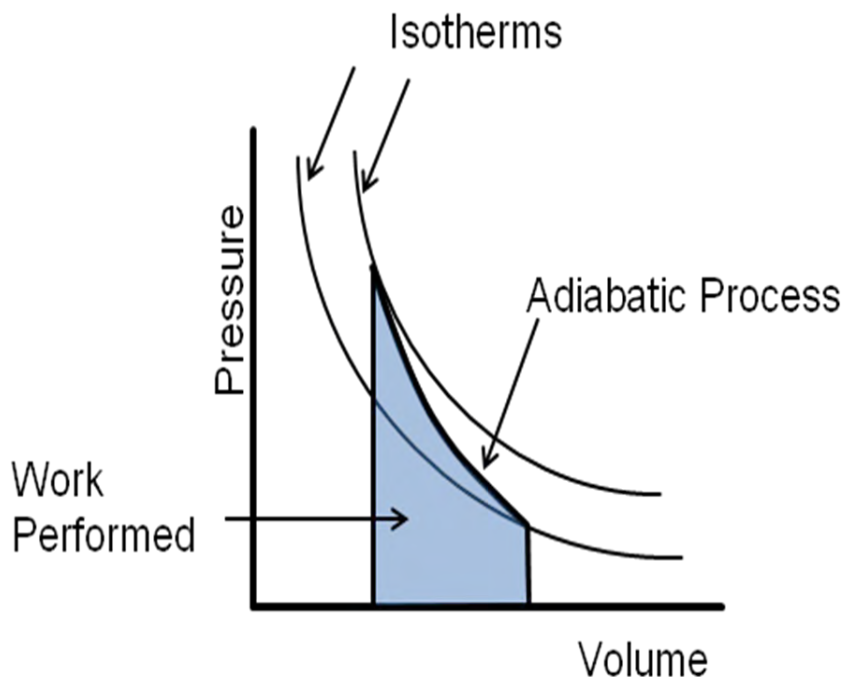
$$p V^{\gamma} = \text{Constant}$$

Eq. 3.4

Where,

γ = Degrees of freedom of the gas molecules; i.e. 7/5 for nitrogen and oxygen.
p = Pressure in SI or US (Imperial) units
V = Volume in SI or US (Imperial) units

Figure 3.1. Work performed in a real gas, adiabatic, thermodynamic process

Figure 3.1 represents a "**real gas**" scenario, where temperature does change to some degree. At an *inversion point*, however, the temperature *does not* change during a throttling process. A real gas tends to behave like a ideal gas as temperature approaches the inversion point. Adiabatic process in an "**ideal gas"** scenario is depicted in Figure 3.2.
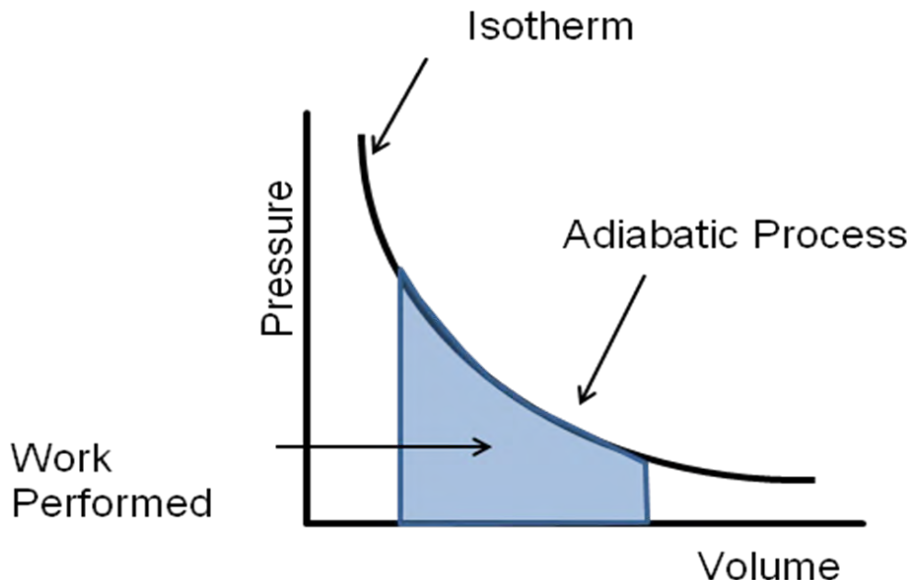
Figure 3.2 Work performed in an ideal adiabatic thermodynamic process

Adiabatic Process Example I - Throttling Process in a Refrigeration System:

Throttling process in a refrigeration system is an example of an adiabatic process that occurs in the expansion valve; where a high pressure liquid system (refrigerant) is allowed to expand to a low pressure liquid, without absorption or release of heat energy.

Adiabatic Process Example II – Compressor Segment of a Refrigeration System:

The compression segment of the refrigeration cycle is an adiabatic as well as an isentropic process. During compression, the refrigerant or the system is compressed from low pressure vapor phase to high pressure vapor phase. No heat is exchanged with the environment during this compression process. The work performed on the refrigerant is negative work, or, **"– W."** Since no heat is released by the system, this negative work, in accordance with Eq. 3.1 and

the law of conservation of energy, is transformed into internal energy of the refrigerant. Compression of vapor, therefore, is an adiabatic process. As discussed later in this text, the compression segment of the refrigeration cycle is not just adiabatic but also an ***isentropic*** process.

Isenthalpic or **Isoenthalpic Process**

An **isenthalpic,** or **isoenthalpic,** process is a thermodynamic process in which no change in enthalpy occurs, or $\Delta h = 0$ or $h_1 = h_2$.

A steady-state, steady-flow process, would be isenthalpic if the following conditions are met:

> 1) The thermodynamic process is adiabatic; meaning, no heat is exchanged with the environment.

> 2) Work is neither performed by the system onto the surroundings nor is it performed by the surroundings onto the system.

> 3) There is no change in the kinetic energy of the system or fluid.

Isenthalpic Process Example - Throttling Process in a Refrigeration System:

Refrigeration system throttling process is an example of an isenthalpic process. See Figure 3.3. Throttling of a high pressure liquid refrigerant to a low pressure liquid phase is an adiabatic process; i.e. no heat is exchanged with the environment. Moreover, no work is done on or by the surroundings, and there is no change in the kinetic energy of the fluid. Note that during the throttling process shown in Figure 3.3, the process adopts a vertical downward path, dropping the pressure precipitously while the enthalpy stays unchanged, thus, rendering the process isenthalpic. In other words, all three requirements or conditions, stated above, for an isenthalpic process are met during the throttling segment of the refrigeration cycle.

Thermodynamic Processes in a Typical Refrigeration Cycle:
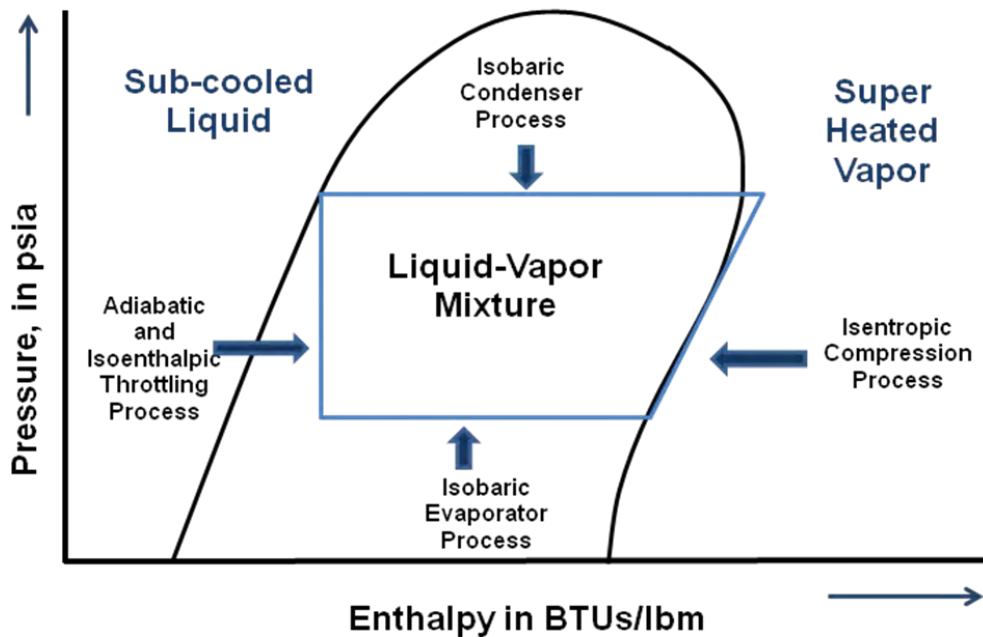
Figure 3.3 Thermodynamic Processes in a Typical Refrigeration Cycle

Other examples of practical isoenthalpic processes include lifting of a relief valve or safety valve on a pressurized vessel. The specific enthalpy of the fluid inside the pressure vessel is the same as the specific enthalpy of the same fluid immediately after it escapes the vessel. In such a scenario, the temperature and velocity of the escaping fluid can be calculated if the enthalpy is known.

In Figures 3.1 and 3.2, an isenthalpic process follows the isotherm line at a specific temperature, and along the isotherm the following relationship between enthalpy, temperature and specific heat holds true:

$$dh = c_p dT = 0$$

Additional examples of isenthalpic process are referenced later in this segment, under the heat engine cycle discussion.

Constant Pressure or Isobaric Process:

An isobaric process is a thermodynamic process in which the pressure remains constant. See Figure 3.4, where the curve represents an isobar. Even though the temperature varies as a function of the entropy in this graph, the pressure stays constant.

## Isobaric Process Example I: Evaporation Stage of a Refrigeration Cycle

Evaporation stage of a refrigeration cycle represents an isobaric process in that the pressure remains constant as the low pressure liquid system evaporates or changes phases from liquid to gaseous by ***absorbing the heat*** energy from the air passing through the heat exchanger. This absorption of heat by the system (refrigerant or the working fluid) from the surroundings (ambient air) is shown in Figure 3.4 as the shaded area under the isobar, between entropies **s_1** and **s_2.**

In an isobaric process:

$$\Delta p \ = \ 0, \text{ and}$$
$$Q = \Delta H$$

The later mathematical statement, $Q = \Delta H$, implies that in this isobaric process, the heat absorbed by the refrigerant during the evaporation phase, results in a net increase in the enthalpy of the refrigerant.
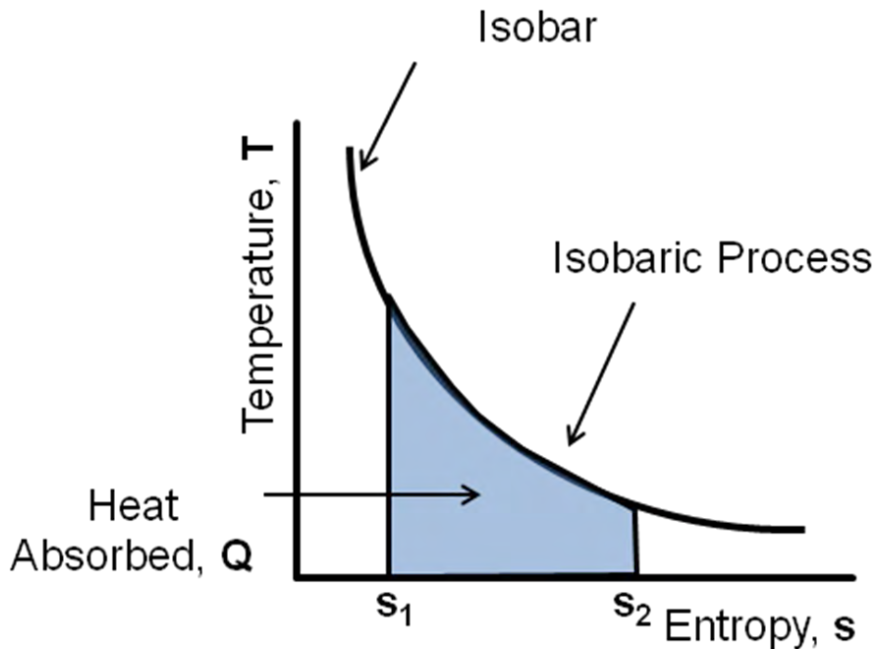
Figure 3.4 Heat absorbed in an isobaric thermodynamic process

Some of the equations with practical applications in closed system isobaric processes are listed below:

$$T_2 = T_1 \left(\frac{\upsilon_2}{\upsilon 1} \right)$$ **Eq. 3.5**

$$\upsilon_2 = \upsilon_1 \left(\frac{T_2}{T_1} \right)$$ **Eq. 3.6**

$$q = h_2 - h_1$$ **Eq. 3.7**

$$q = c_p (T_2 - T_1)$$ **Eq. 3.8**

$$q = c_v (T_2 - T_1) + p(\upsilon_2 - \upsilon_1)$$ **Eq. 3.9**

$$W = p(\upsilon_2 - \upsilon_1)$$ **Eq. 3.10**

$$W = R(T_2 - T_1)$$ **Eq. 3.11**

Where,

q = Heat per unit mass or, *Q/m*
= Total Heat/Unit Mass, in BTU/lbm or kJ/kg
p = Pressure in lbf/ft^2 or Pa
v_1 = Initial specific volume in ft^3/lbm or m^3/kg
v_2 = Final specific volume in ft^3/lbm or m^3/kg
h_1 = Initial enthalpy in BTU/lbm or kJ/kg
h_2 = Final enthalpy in BTU/lbm or kJ/kg
T_1 = Initial Temperature, in °F, °C, °K, R
T_2 = Final Temperature, in °F, °C, °K, R
R = Specific gas constant, in ft-lbf/lbm-°R or kJ/kg.°K
W = Specific work in ft-lbf/lbm or kJ/kg. W also represents total work in ft-lbf or kJ

Note that Eq. 3.5 and Eq. 3.6 are derived from the ideal gas law, as stated in form of Eq. 3.12 below, with pressure held constant in an isobaric process.

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

Eq. 3.12

Isobaric Process Example II: Isobaric Segments of an Ideal Cycle Heat Engine

As shown in Figure 3.5, ideal heat engine cycle segments represented by processes A-B and C-D are isobaric processes because during these two processes within the heat engine cycle, the pressure remains constant as the volume expands and contracts, respectively. The working fluid performs work on the surroundings as it expands, from point A to B. This work is considered *positive*. The surroundings perform work on the system, or working fluid, from point C to D; resulting in *negative* work. As explained in greater detail under the heat cycle section, this negative work is in the form of the condensate pump performing work on the condensed vapor as it is pressurized to saturated liquid phase.
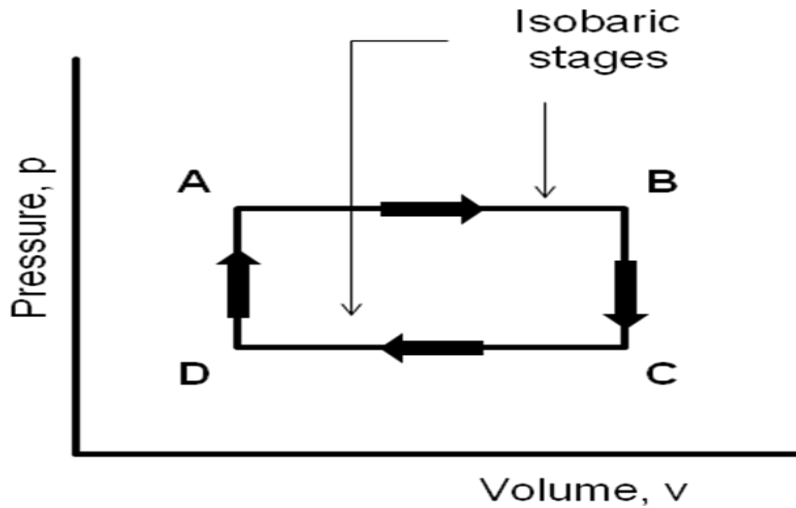
Figure 3.5 Isobaric Process in an Ideal Heat Engine Cycle

Constant Temperature or Isothermal Process:

An isothermal process is a thermodynamic process in which the temperature stays constant. In isothermal processes, there is no change in internal energy because internal energy is directly related to temperature. This is validated by Eq. 3.17. Furthermore, as stipulated by Eq. 3.18, there is no change in enthalpy.

Some of the equations with practical applications in closed system isothermal processes are listed below:

$$T_2 = T_1 \qquad \text{or}, \qquad \Delta T = 0 \qquad\qquad \textbf{Eq. 3.13}$$

$$p_2 = p_1 \left(\frac{\upsilon_1}{\upsilon_2} \right) \qquad\qquad \textbf{Eq. 3.14}$$

$$\upsilon_2 = \upsilon_1 \left(\frac{p_1}{p_2} \right) \qquad\qquad \textbf{Eq. 3.15}$$

$$q = W \qquad\qquad \textbf{Eq. 3.16}$$

$$u_2 - u_1 = 0 \qquad\qquad \textbf{Eq. 3.17}$$

$$h_2 - h_1 = 0 \qquad\qquad \textbf{Eq. 3.18}$$

$$Q = nR^*T \ln\left(\frac{p_1}{p_2}\right) \qquad \textbf{Eq. 3.19}$$

Where,

> q = Heat per unit mass or, Q/m = Total Heat/Unit Mass, in BTU/lbm or kJ/kg
>
> Q = Total heat in BTU or kJ. Q also denotes molar heat in BTU/lbmol or kJ/kmol
>
> p_1 = Initial Pressure in lbf/ft^2 or Pa
>
> p_2 = Final Pressure in lbf/ft^2 or Pa
>
> v_1 = Initial specific volume in ft^3/lbm or m^3/kg
>
> v_2 = Final specific volume in ft^3/lbm or m^3/kg
>
> h_1 = Initial enthalpy in BTU/lbm or kJ/kg
>
> h_2 = Final enthalpy in BTU/lbm or kJ/kg
>
> T_1 = Initial Temperature, in °F, °C, °K, R
>
> T_2 = Final Temperature, in °F, °C, °K, R
>
> n = Number of moles, in lbmole or kmol
>
> R = Specific gas constant, in ft-lbf/lbm-°R or kJ/kg.°K
>
> R^*= Universal gas constant, in ft-lbf/lbmole-°R or kJ/kmol-°K
>
> W = Specific work in ft-lbf/lbm or kJ/kg. W also represents total work in ft-lbf or kJ

Isothermal Process Example I: Steam Generation Process

The, latent, water evaporation stage in the steam generation process is an isothermal process because the temperature of the water and saturated vapor remains constant until all evaporation is concluded.

The work "**W**" performed by, or on, an isothermal system, is shown in Figure 3.6. Where work $\mathbf{W} = \mathbf{W}_{X \rightarrow Y}$.

And,

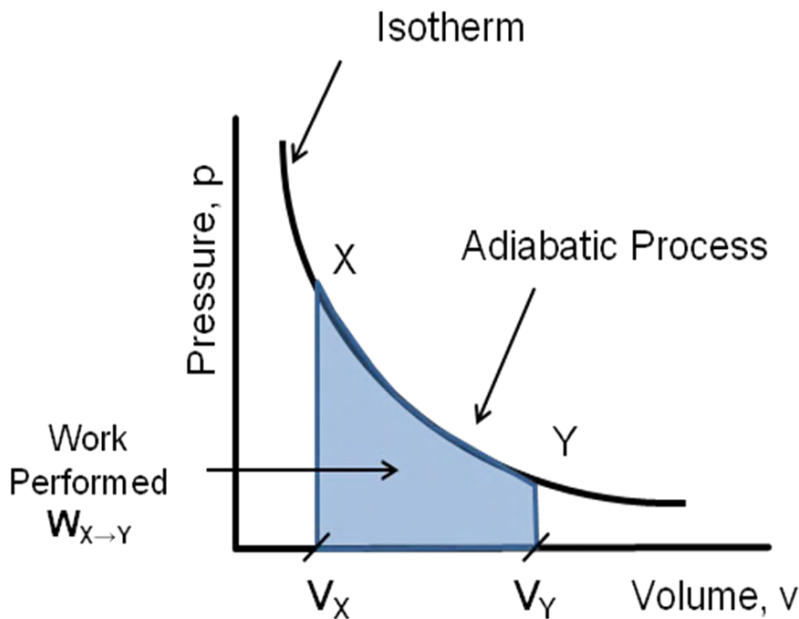$$W_{X \rightarrow Y} = -\int_{V_X}^{V_Y} p.dV \qquad \textbf{Eq. 3.20}$$

Figure 3.6 Isothermal Process in a Heat Engine Heat Cycle

Example Problem 3.1:

Consider the collision of the cast iron block with the compressed air filled shock absorbing system described in Case Study 2, part (e), Segment 1. A cooling jacket is installed on the cylinder to maintain the temperature constant, at 20°C. For simplicity, assume that 1.3 kg of air is present in the shock absorbing system cylinder. Calculate the amount of heat, in BTU's, the cooling jacket would need to remove each time a block is stopped.

Solution:

According to Eq. 3.19, the heat removed to maintain constant temperature in an isothermal compression process would be:

$$Q = nR^*T \ln\left(\frac{p_1}{p_2}\right)$$

Where,

Q = Total heat removed

n = Number of kmols of gas involved =?

R^* = Universal Gas Constant = 8.314 kJ/kmol.°K or, 1545 ft-lbf/lbmole.°R. See Table 3.1 below.

p_1 = Initial pressure = 101.3 kPa = 1 Bar

p_2 = Final Pressure = 202.6 kPa = 2 Bar
T = 20 °C + 273 = 293°K

Since the number of kmols is not given, it needs to be derived using given mass of air, i.e. 1.3 kg and the molecular weight listed in Table 3.1.

The molecular weight of air, from Table 3.1 is 28.97 kg/kmol.

Therefore,

n = Number of kmols of air = 1.3 kg / 28.97 kg/kmol
= 0.0449 kmols

Then, from Eq. 3.19:

$$Q = (0.0449 \, kmol)(8.314 \, kJ/kmol°K)(293°K)\ln\left(\frac{1}{2}\right)$$

= **-76 kJ of heat removed.**

Since 1kJ = 0.95 BTU,

$Q_{\text{removed, in BTU's}}$ = **(-76 kJ).(0.95 BTU/kJ)**
= **-71.2 BTUs**

Gas	Molecular Weight	Gas	Molecular Weight
Acetylene, C_2H_2	26.04	Natural Gas	19
Air	28.966	Nitric Oxide, NO_2	30.006
Ammonia (R-717)	17.02	Nitrogen, N_2	28.0134
Argon, Ar	39.948	Nitrous Oxide	44.012
N-Butane, C_4H_{10}	58.12	Oxygen, O_2	31.9988
Iso-Butane	58.12	Ozone	47.998
Carbon Dioxide, CO_2	44.01	Propane, C_3H_8	44.097
Carbon Disulphide	76.13	Propylene	42.08
Carbon Monoxide, CO	28.011	R-11	137.37
Ethane, C_2H_6	30.07	R-12	120.92
Ethyl Alcohol	46.07	R-22	86.48
Ethylene, C_2H_4	28.054	R-114	170.93
Helium, He	4.02	R-123	152.93
Hydrogen, H_2	2.016	R-134a	102.03
Hydrogen Chloride	36.461	R-611	60.05
Hydrogen Sulfide	34.076	Sulfur	32.02
Methane, CH_4	16.044	Sulfur Dioxide	64.06
Methyl Alcohol	32.04	Sulfuric Oxide	48.1
Methyl Butane	72.15	Water Vapor/Steam,	18.02

Table 3.1 – Molecular Weights of Common Gases Associated with Combustion Reactions and Byproducts

Constant Volume Process:

A constant volume process is also referred to as an isometric process or an iso-volumetric process. In a constant volume thermodynamic process, the volume of a closed system remains constant while other parameters, i.e. pressure, internal energy and temperature vary.

Pressure energy, work, volume and pressure are related by the following equation:

$$\Delta W = P\Delta V, \text{ where P is pressure}$$

Since the volume is constant in an isometric process,

$$\Delta V = 0$$

And

$$\Delta W = 0$$

Therefore, in an isometric or constant volume process, by application of First Law of Thermodynamics:

$$Q = \Delta U$$

In other words, in an isometric or isochoric process, the heat added to the system is transformed into the higher level of the system's internal energy "U."

Constant Volume Process Example I: Superheated steam generation in a "rigid" constant volume boiler.

Most boilers consist of rigid vessels, tanks, channels or tubes. Since these systems are rigid, as the water is boiled, evaporated into saturated steam, and heated further into superheated steam, the temperature and pressure of the water or steam increase but the volume remains constant. There are two basic approaches to boiler design: (1) Water Tube Boiler and (2) Fire Tube Boiler. The water tube boilers are more common. Figure 3.7 illustrates the fundamental design concept for Water Tube Boilers.
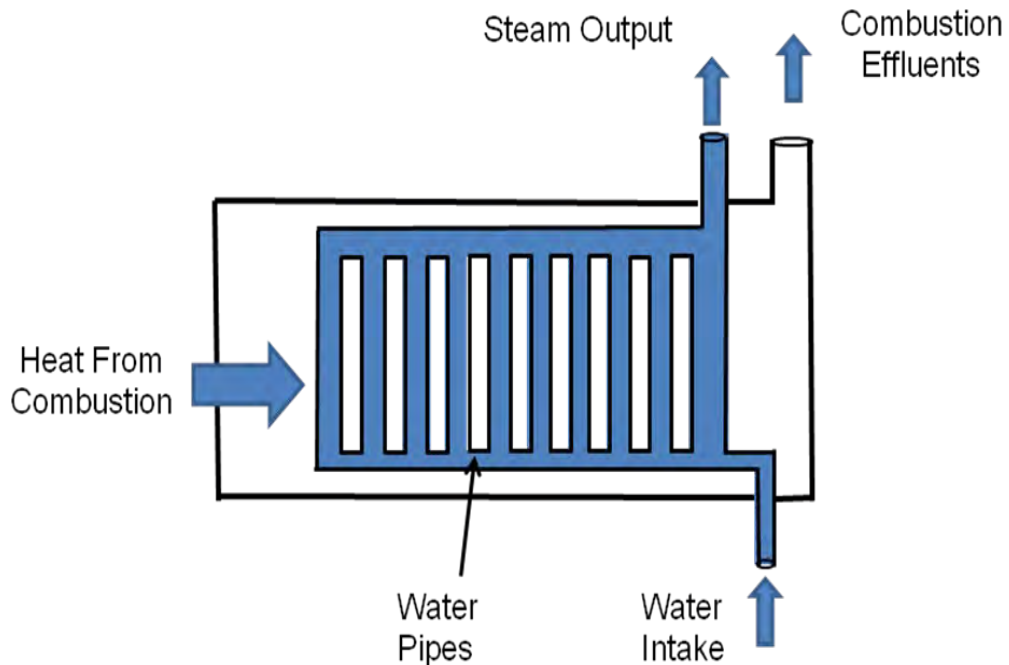
Figure 3.7, Fire-Tube Boiler Design and Operation Concept Diagram

Constant Volume Process Example II: Ideal Heat Engine

Processes or paths D-A and B-C in an ideal heat engine cycle, as shown in Figure 3.5 Pressure–Volume diagram, provide another example of constant volume or isometric thermodynamic process. In paths D-A and B-C, while the pressure increases and decreases, respectively, the volume stays constant.

Isentropic or Constant Entropy Process:

Isentropic process, in a thermodynamic system, is a process in which the entropy of the system stays constant. Any reversible adiabatic process is an isentropic process.

Isentropic Process Example I: Ideal Heat Engine – Carnot Cycle

Process paths D´-A´ and B´-C´, as shown in the Temperature-Entropy diagram for a Carnot Cycle Heat Engine in Figure 3.8, are examples of isentropic process. Note: The Carnot Cycle is explained later in this segment.

In paths D´-A´ and B´-C´, while the temperature rises and drops, respectively, the entropy "**s**" stays constant.
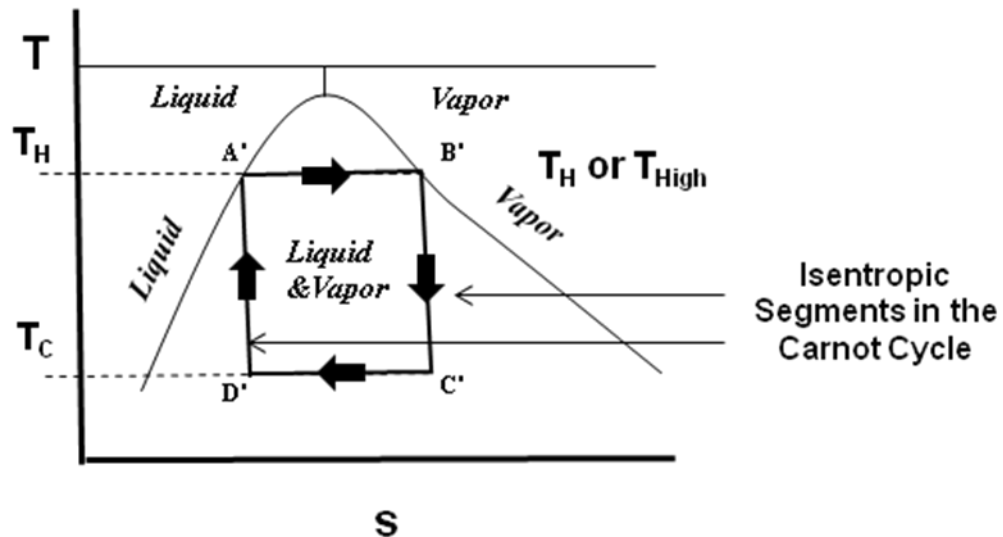
Figure 3.8. Isentropic Processes in a Carnot Cycle, an Ideal Cycle Heat Engine

Throttling Process and Inversion Point:

Throttling process in a thermodynamic system is an adiabatic process which consists of a significant pressure drop but no change in the system enthalpy. Furthermore, in a throttling process, no heat is exchanged with the surrounding and no work is performed on or by the system. Since the enthalpy in a throttling process stays constant, a throttling process is also an isenthalpic (constant enthalpy) process.

In a throttling process:

$$p_2 < < p_1$$

$$\Delta H = 0, \text{ and } \Delta h = 0$$

Where,

ΔH = Change in "absolute" enthalpy, measured in BTUs, Joules (J) or kilo Joules (kJ)

Δh = Change in specific enthalpy, measured in BTUs/lbm, Joules (J)/gram or kilo Joules (kJ)/kilogram (kg)

p₂ = Final, lower, pressure

p₁ = Initial, higher, pressure

In ideal gas systems, throttling processes are constant temperature processes. In real gas scenarios, temperature change does occur when the gas is throttled. However, for every real gas, under a given set of conditions, there is a temperature point at which no temperature change occurs when the gas is throttled. This temperature is called an ***inversion point.*** For air, maximum inversion temperature is 603 °K.

Thermodynamic Equilibrium:

A Thermodynamic system is said to be in equilibrium when it is in a thermal, chemical, mechanical, convectional and radiative state of balance. A thermodynamic system in equilibrium experiences no thermal, chemical, mechanical, radiative and convectional changes when isolated or insulated from the surroundings.

Quasistatic or Quasiequilibrium Process:

Some thermodynamic systems are in equilibrium at the beginning of a process and are in equilibrium toward the end of a process. However, these systems may deviate from equilibrium at interim points of the process. Such processes are referred to as *quasistatic* **or** *quasiequilibrium processes.* Such processes are said to constitute infinitesimal steps. Since the property changes in each of these steps are small, for all intensive purposes, these steps are assumed to represent short equilibrium phases.

Polytropic Process:

Polytropic processes pertain to gases and are processes that function in accordance with the polytropic equation of state:

$$(\upsilon_1)^n \, p_1 = p_2 (\upsilon_2)^n$$

Eq. 3.21

Where, "n" is a polytropic exponent and is an intrinsic property of the equipment and not the gaseous system. For instance, the polytropic exponent "n" for air compressors ranges from 1.25 to 1.3.

Note the difference between the polytropic process equation **Eq. 3.21** and ideal gas law equation **Eq. 3.15a.**

$$\upsilon_2 = \upsilon_1 \left(\frac{p_1}{p_2} \right)$$

Eq. 3.15.

Equation 3.15, which is also a mathematical representation of an ideal gas law referred to as the Boyles Law, can be rearranged and written as:

$$\upsilon_1 p_1 = p_2 \upsilon_2$$

Eq. 3.15a.

A comparison between **Eq. 3.15a** and **Eq. 3.21** reveals that a salient difference between the behavior of an ideal gas, under ideal conditions, and ideal gas (air) operating under specific equipment specification is that, under specific equipment constraints, the exponent "n" of volumes υ_1 and υ_2 is not equal to 1. Hence, "**n**" is used as an exponent for the volume in **Eq. 3.21**.

Reversible Process:

A thermodynamic reversible process is a process that changes the state of a system in such a way that the net change in the combined entropy of the system and its surroundings is zero. The system and the surroundings *can* be restored to their initial states at the conclusion of a reversible process. No heat is wasted in a reversible process, therefore, the machine or engine's efficiency is maximized.

One of the attributes of a reversible process can be stated, mathematically, as follows:

$$\Delta S = 0$$

Irreversible Process:

A thermodynamic process that is not reversible is referred to as an irreversible process.

In addition to the fact that heat is or can be wasted in an irreversible process, there is a net change in entropy of the system. In other words:
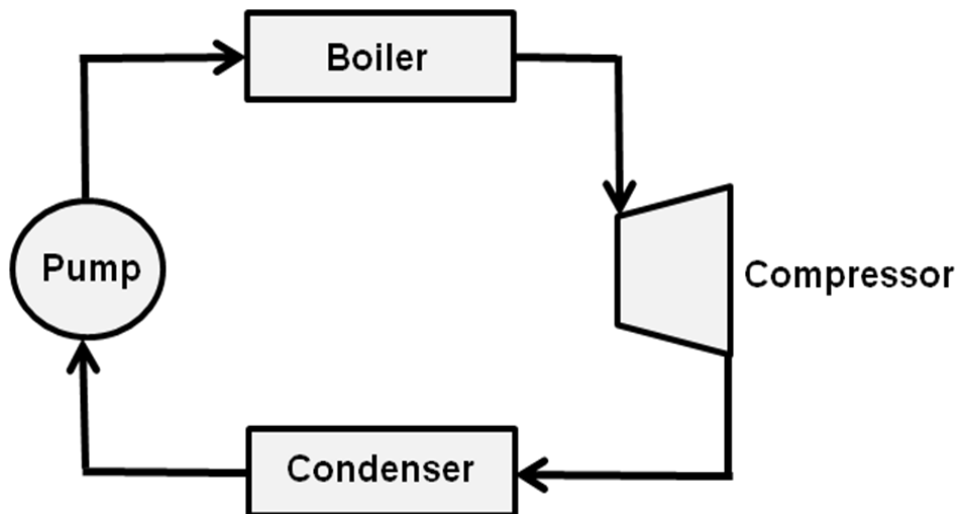
$$\Delta S \neq 0 \mid \text{Irreversible}$$

Ideal Heat Engine, Ideal Heat Engine Cycle and Energy Flow

Since the purpose of most engines is to convert one form of energy to another and perform work, an ideal heat engine's function can be simplified and understood through evaluation of the heat/energy flow diagram in Figure 3.9.
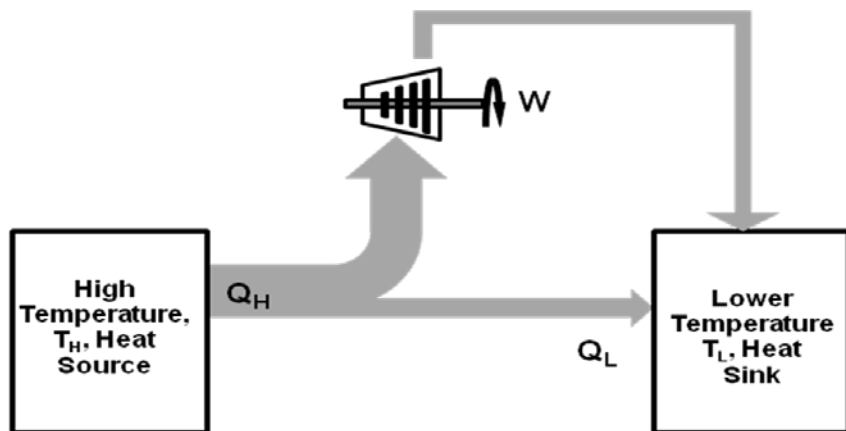
Figure 3.9 Heat Engine Energy Flow Diagram

Figure 3.10 Heat Engine Process Flow Diagram

As depicted in Figure 3.9, a heat engine performs the conversion of heat energy to mechanical work, driven by the temperature gradient between the higher heat content heat source and a lower heat content point referred to as the heat sink.

Ideal heat engine cycle and the flow of heat and energy in an ideal heat engine can be illustrated better through examination of the Heat Engine Energy Flow Diagram in Figure 3.9, and Heat Engine Process Flow Diagram in Figure 3.10, conjunctively.

As the heat energy is transferred from the heat source to a heat sink, through a mechanical device, such as a turbine, a substantial portion of the heat energy is transformed into mechanical energy or work. This is analogous to the flow of electrical current in an electrical circuit where current is driven by the electromotive force, or voltage difference between the positive terminal and the negative terminals of the voltage source. In the analogous electrical current scenario, when the current is routed through an electromechanical device like a motor, it ends up converting the electrical energy into mechanical energy, in the form of work.

The transformation of heat energy to mechanical work through a heat engine is also analogous to the energy transformation that occurs in hydroelectric projects. In hydroelectric power generating systems, of course, the elevation head, or potential energy stored in the elevated water of the reservoir, is converted into the kinetic head or kinetic energy as the water is allowed to flow to the turbine. This kinetic energy is converted into mechanical energy of the hydroelectric turbine. The mechanical energy of the hydroelectric turbine, similar to a steam turbine, is then converted into electrical energy, or electric power, through electromagnetic transformation in the electric power generator.

A steam engine is a form of heat engine with the specific purpose of converting heat energy, derived through the combustion of some type of fuel, into mechanical energy or mechanical work. Of course, the complete process involves the transfer of heat energy from combustion of fuel to water. In the case of steam based power generating plants, the water, through sensible and latent process, is typically converted to high enthalpy, superheated steam. Water in such a process is referred to as the ***working fluid.*** The heat energy

contained in the superheated steam is then used to turn the turbine. The mechanical energy of the turbine is converted into electrical energy (kWh) through the coupled shafts of the turbine and the electric power generator. In the last stage, conversion of mechanical energy to electrical energy involves the rotation of generator's rotor within the magnetic field of the armature, thus inducing current flow in the armature or stator of the electric power generator.

Steam engines are commonly designed to operate in a heat cycle referred to as the ***Rankin Cycle.*** A ***Rankin Engine*** is simply an engine, or an energy conversion system, in which the working fluid experiences or follows a Rankin cycle. The vapor cycle pertaining to a Rankin cycle is a, relatively, simple and practical vapor cycle. Therefore, in this segment, we will explore a basic Rankin cycle in detail, followed by a brief description of other variations of the Rankin cycle and an introduction to the Carnot cycle.

Rankin Cycle engines generate, approximately, 80% of all of the electric power consumed in the world. This includes the nuclear fission, fossil fuel, biomass and geothermal based electric power generation. Some notable renewable methods for generation of electric power, i.e. CSP, Concentrated Solar Power, geothermal power generation systems and biomass combustion systems, to name a few, employ the Rankin Cycle for conversion of heat energy into electrical energy. Figure 3.11 and Figure 3.12 show pictures and diagrams of examples of solar and geothermal projects.


Figure 3.11 Concentrated Solar Power Tower – Solucar, Spain Project.
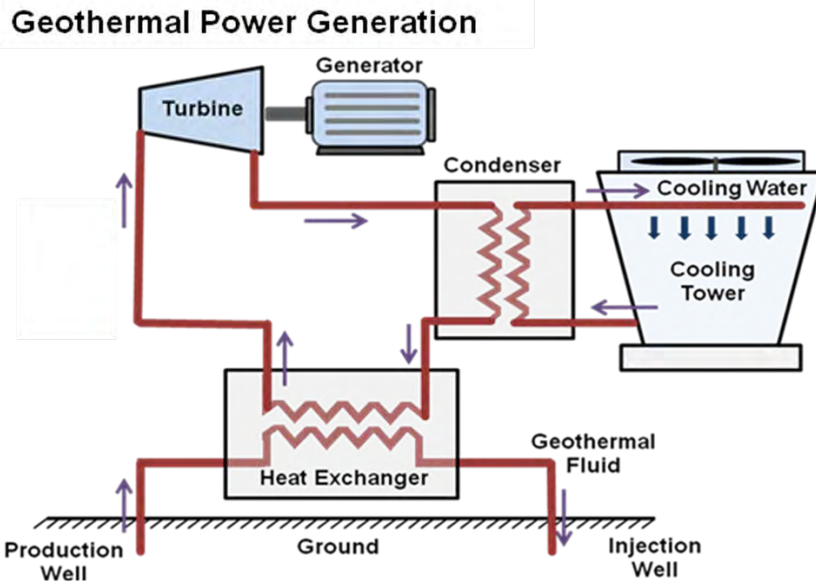
**Figure 3.12 Geothermal Electrical Power Generation Process.**

Water is typically used as the working fluid in the most Rankin Cycle Engines, because of its favorable properties, such as, its relative chemical inertness; water is neither reactive nor toxic. Water is also abundant, accessible and available at reasonable costs. Figure 3.13 represents a pictorial or physical view of the Rankin cycle system. Figure 3.13 is based on symbolic mechanical drawings of the various components of this Rankin Engine.
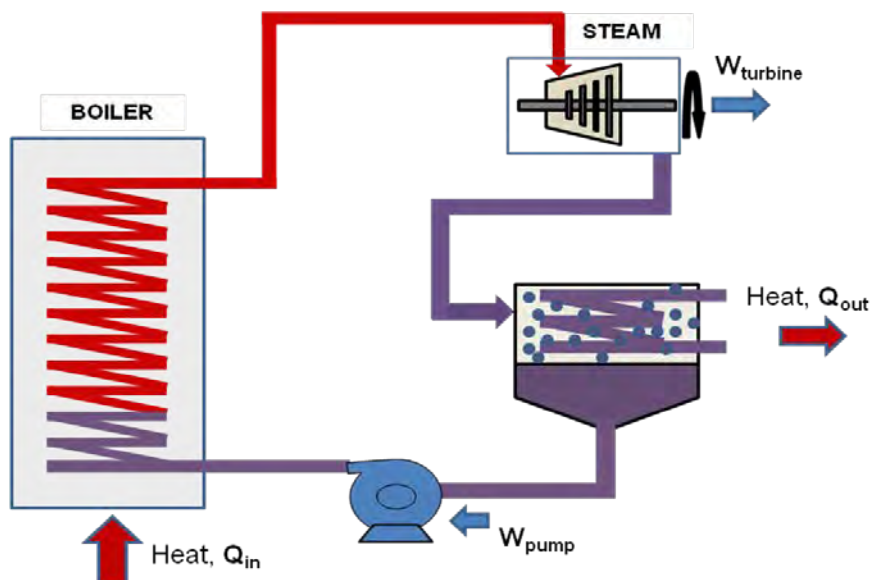
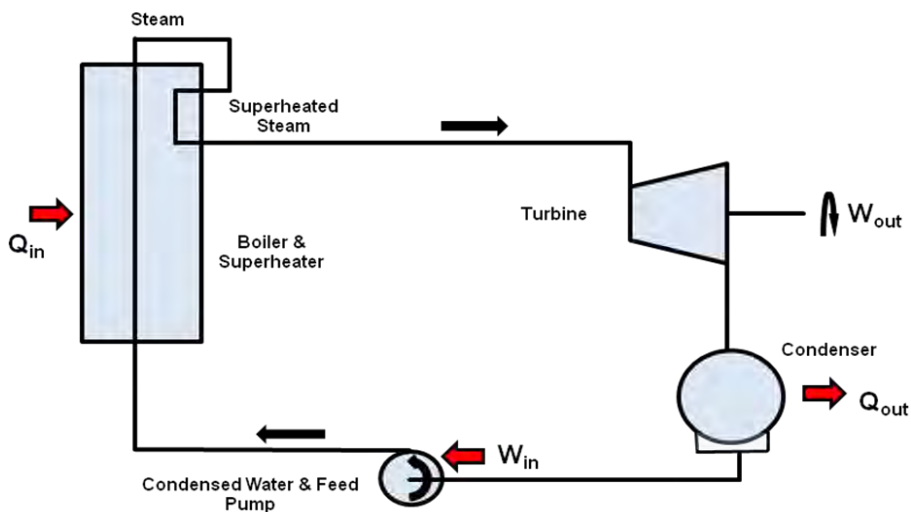**Figure 3.13 Heat cycle in a heat engine without superheat.**

As shown in Figures 3.14 and 3.15, in Rankin Cycle Engines, water is used in a closed loop configuration. Figure 3.14 and Figure 3.15 depict the Rankin Heat Engine and the associated vapor cycle. Figure 3.14 shows the flow of water and vapor in a *single line diagram* form utilizing conventional symbols.
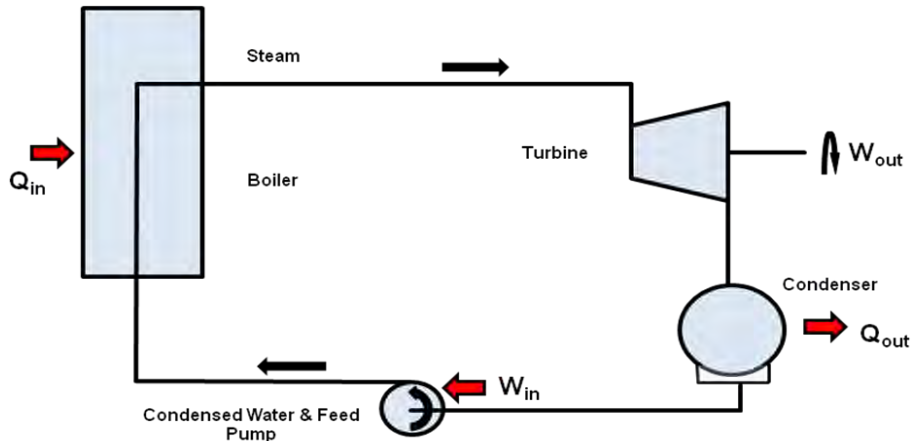
Figure 3.14 Heat cycle in a heat engine without superheat.

While Figure 3.14 represents a one line diagram of the Rankin cycle system that utilizes *a simple* water to vapor conversion stage, Figure 3.15 represents a Rankin cycle system that utilizes a water to vapor conversion stage coupled with a *vapor superheating stage*.

Figure 3.15 Heat cycle in a Rankin Engine with Superheat

Since the Rankin Cycle with the superheat represents a more common and practical Rankin cycle, we will explore it in depth in an effort to understand the process flow in Rankin Heat Engines, in general. Variations of the Rankin Engine, or Rankin Cycle will be described, briefly, later in this segment.

Examination of Figure 3.15 shows that a basic Rankin Cycle, or a basic Rankin Engine consists of the following major components:

1) Boiler equipped with auxiliary superheating function.
2) Turbine
3) Condenser
4) Pump

Before we embark on a detail discussion of the Rankin Cycle, let us define the four major Rankin Cycle components stated above.

1) **Boiler with Superheating Function:** A boiler, sometimes referred to as a steam generator is essentially a system designed to heat the feed water returned from the condenser when a heat engine recoups the condensate through a condenser in lieu of releasing it into the environment as effluent. This water is heated up to the saturated state through introduction of heat from the combustion burners. The saturated water is further heated to convert the saturated water into saturated steam. The superheating segment of the boiler converts the saturated steam and some of the lower temperature superheated steam into higher temperature superheated steam. The steam is heated to high temperature superheated steam in order to ensure that vapor does not condense in the turbine. When steam condenses into liquid phase in the turbine segment of the heat engine, it tends to reduce the efficiency or energy output of the turbine, and it tends to cause pitting, corrosion and accelerated deterioration of the turbine blades.

2) **Turbine**: A turbine, in essence, converts the superheated steam's enthalpy, or heat content, into positive work, or work performed by the system on the surroundings. From basic function and operating principle perspective, a steam turbine is similar to a hydroelectric turbine or a wind turbine. In all three instances, the energy contained in the fluid, gaseous or liquid is converted into mechanical energy or *brake horsepower*.

There are two major categories or types of steam turbines that can be differentiated on the basis of their operation. These two categories are as follows:

A. Reaction Turbine

B. Impulse Turbine

Reaction Turbine: A reaction turbine is constructed in a form of a drum equipped with nozzles or reaction jets. These jets or nozzles, as shown in Figure 3.16, are located around the circumference of the drum. As the steam exits from the nozzles at a certain velocity **V** and mass flow rate **ṁ**, a force equal and opposite to the force F, below , is produced rotating the drum in the opposite direction. In Figure 3.16, this force rotates the drum in the counter clockwise direction. The formula for this force is stated in the form of Eq. 3.22

$$\mathbf{F = \quad \dot{m} \cdot \Delta V} \qquad\qquad \textbf{Eq. 3.22}$$
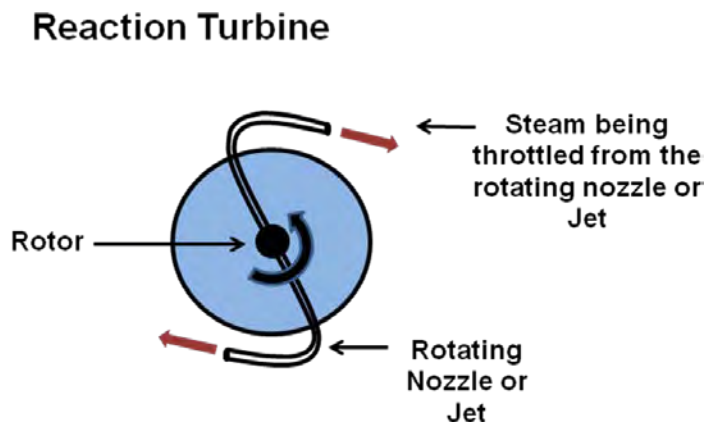
Reaction Turbine

Figure 3.16 Reaction Turbine Design and Operating Principle

Impulse Turbine: An impulse turbine system consists of stationary nozzles and vanes appointed on a wheel as shown in Figure 3.17. The high pressure high enthalpy steam released in a throttling process from the opposing nozzles impacts the vanes in a coupled moment fashion. This *coupled* moment, as depicted in Figure 3.18, rotates the wheel, thus transferring power to the wheel shaft as brake horsepower.
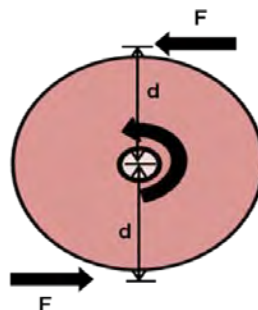
A coupled moment, or simple a couple is comprised of two equal and opposite forces. The sum of these forces, as stated in Eq. 3.23, is zero.

$$\sum F = 0$$

Eq. 3.23

The coupled moment is shown in Figure 3.18 and represented mathematically in Eq. 3.24.
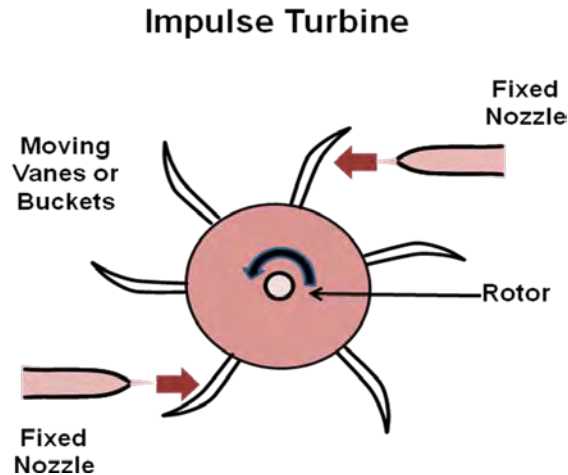
Impulse Turbine

Figure 3.17 Impulse Turbine Design Principle

Figure 3.18. Couple in an Impulse Turbine

$$Couple = 2.\tau = 2(F.d)$$

Eq. 3.24

Where,

τ is the torque

> **F** is the magnitude of the equal and opposite forces
> **d** is the moment arm for each force, spanning from the center point of the vane to the center of the turbine wheel.

The throttling process that steam undergoes in the impulse turbine nozzles, as well as in the reaction turbine nozzles, is an adiabatic process; no heat is exchanged between the system and the environment. This can be physically explained on the basis of the fact that due to the high velocity of steam and short contact period between the steam and the nozzles, a negligible amount of heat is transferred from the steam to the nozzles. In other words, the enthalpy remains unchanged; **h₁ = h₂** and Δh = 0. This tenet is supported by the Bernoulli's law with appropriate assumptions and simplification.

3) **Condenser:** A condenser condenses the lower enthalpy vapor into water by facilitating extraction of heat "Q" from the saturated vapor. As explained in Segment 2, when saturated vapor loses heat, it condenses into saturated liquid phase, or water. In that respect, a condenser can be referred to as a heat exchanger. A condenser is considered to be a special purpose non-adiabatic heat exchanger. A condenser is non-adiabatic because heat flows out of it as the working fluid, or vapor, is cooled. The heat flow rate out of a condenser is defined mathematically in Eq. 3.25. Since heat is lost by the vapor in the condenser, the final enthalpy will be lower than the initial enthalpy. In other words, (h₂ – h₁) would be negative.

$$\dot{Q} = \dot{m}(h_2 - h_1) \qquad \textbf{Eq. 3.25}$$

In Eq. 3.25:

$\dot{Q}$ represents the heat flow rate in BTU's per second, or kJ/second.

$\dot{m}$ represents the mass flow rate in lbm per second or kg/second.

h₂ is the final enthalpy.

h₁ is the initial enthalpy.

4) **Feed Pump:** The feed pump simply receives the condensed water from the condenser, pressurizes it and transfers it to the boiler.

Process Flow in a Rankin Cycle with Superheat

In order to analyze and understand the processes or stages in the superheat equipped Rankin Cycle, the one line schematic from Figure 3.15 has been redrawn with appropriate stage or process annotations as shown in Figure 3.19. These stages or processes are further addressed through pressure "**p"** vs. specific volume **"υ,"** temperature "**T**" vs. entropy "**s**," and enthalpy **"h"** vs. entropy "**s**" graphs; these graphs are depicted in Figure 3.20, Figure 3.21 and Figure 3.22, respectively.
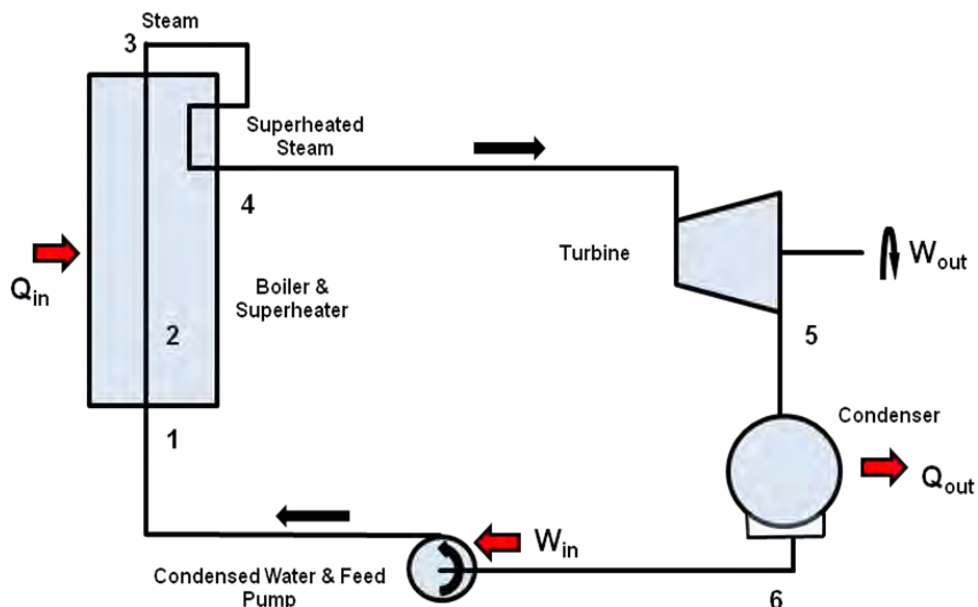
Figure 3.19 Heat Cycle in a Rankin Engine with Superheat

The thermodynamic process flow stages in a superheated Rankin cycle, as shown in Figure 3.19, are numbered 1 through 6. In Figure 3.20, Figure 3.21 and Figure 3.22, these six stages are described in terms of the interplay between pressure (**p**), specific volume (υ), temperature (**T**), enthalpy, (**h**) and entropy (**s**).

Process Flow from Point 1 to 2:

Point 1 lies in the sub-cooled water realm on all three graphs. In other words, it is located distinctly to the left of the bell shaped saturation curves on all three graphs. On the other hand, point 2 lies directly on the saturation curve on all three graphs. This means that at point 2, the water is in saturated liquid state. Therefore, as heat is added to the subcooled water in the boiler, it transitions from point 1 to point 2 and changes states from subcooled form to saturated water form. As shown in the pressure (**p**) vs. specific volume (**υ**) diagram (Figure 3.20), this process is **isobaric**. Of course, since *heat is being added* to the system or the working fluid, the process flow from point 1 to point 2 is *non-adiabatic.*

As we examine the temperature vs. entropy graph, in Figure 3.21, we note that the process flow from point 1 to point 2 involves a **"sensible"** rise in temperature (**T**) accompanied by an increase in the entropy (**s**).

The transition from point 1 to point 2, from the enthalpy (**h**) and entropy (**s**) perspective, can be observed in Figure 3.22. This transition entails a small increase in the enthalpy and a more pronounced rise in the entropy.
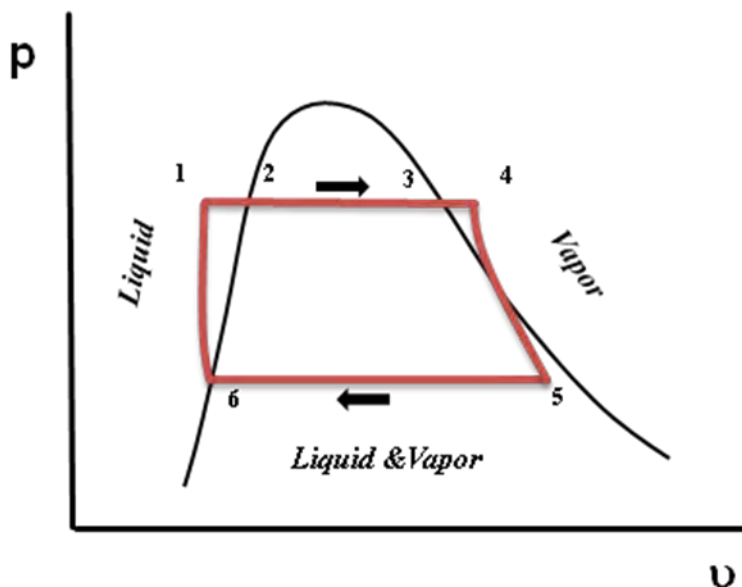

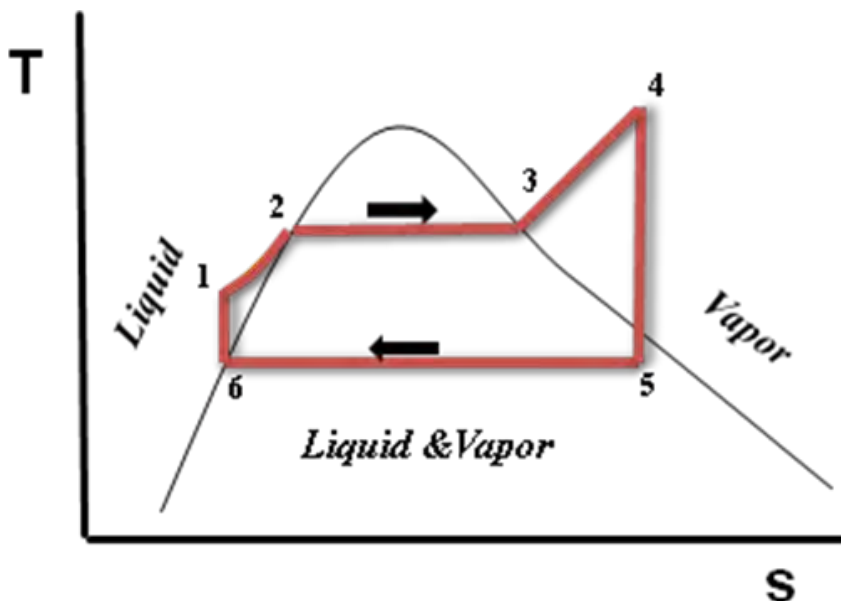**Figure 3.20 Heat Cycle in a Rankin Engine with Superheat, p vs. υ.**

Process Flow from Point 2 to 3:

Point 2 lies directly on the saturation curve on all three graphs. As stated earlier, the water is in saturated liquid state at point 2. By definition, addition of *any* heat to the water in process stage 2 would result in evaporation.

Therefore, as heat is added to the saturated water in the boiler, it transitions from point 2 to point 3 and changes states from saturated water to saturated water vapor. As shown in the pressure **p** vs. specific volume υ diagram (Figure 3.20), this process is *isobaric*.

As we examine the temperature **T** vs. entropy **s** graph, in Figure 3.21, we note that the process flow from point 2 to point 3 is an *isothermal* process coupled with an *increase* in the entropy.

The transition from point 2 to point 3, from the enthalpy **h** and entropy **s** perspective, as graphed in Figure 3.22, shows that the addition of heat in this stage not only raises the entropy but also raises the enthalpy as the heat is added to change states from saturated water to saturated vapor.

Figure 3.21 Heat Cycle in a Rankin Engine with Superheat, T vs. s.

Process Flow from Point 3 to 4:

Point 3 lies directly on the saturated vapor curve on all three graphs. The water is in saturated vapor state at point 3. By definition, *removal* of *any* heat from saturated vapor would result in condensation of the saturated vapor into liquid water.

However, as heat is *added* to the saturated vapor in the boiler, it transitions from point 3 to point 4 and changes states from saturated vapor to superheated vapor. All three graphs show that as additional heat is added to the saturated vapor at point 3, the vapor diverges off the saturation curve and into the superheated steam region. The pressure **p** vs. specific volume υ graph in Figure 3.20 shows that this addition of heat that drives the water into the superheated state is also *isobaric.* Also, note that the high pressure of the superheated steam that is introduced into the turbine, is established in the preceding stages of the heat engine cycle.

The temperature **T** vs. entropy **s** graph, in Figure 3.21, shows that the heat added in this stage escalates the temperature significantly; accompanied by a notable rise in the entropy. The increase in entropy can be observed in the enthalpy vs. entropy graph in Figure 3.22, as well.

Process Flow from Point 4 to 5:

The transition from point 4 to point 5 takes place in the turbine. This is the process stage in which heat and the mechanical energy is delivered from the superheated steam to the turbine blades. That energy is transferred to the turbine shaft in the form of work. The work performed on the turbine blades $W_{Turbine}$, is represented mathematically by Eq. 3.28, in per unit mass form.

Point 4 lies clearly in the superheated steam region as depicted in all three graphs in Figures 3.20 through 3.22. However, as the superheated steam transfers its energy to the turbine, its temperature, enthalpy and pressure drop to levels represented by point 5 on the pressure (**p**), specific volume (υ), temperature (**T**), enthalpy, (**h**) and entropy (**s**) diagrams.

As exhibited in the enthalpy vs. entropy and the temperature vs. entropy graphs, the entropy remains constant in this stage, making this stage

isentropic. Note that the heat is neither added nor removed from the superheated vapor – the working fluid – in this stage. Therefore, this stage is not just isentropic but also adiabatic. Of course, this is based on the assumption that the turbine is well insulated.

The pressure, specific volume, temperature, enthalpy and entropy diagrams show that, after delivering its energy to the turbine, the vapor and liquid mixture has a significantly lower temperature, enthalpy and pressure, and is therefore no longer capable of performing useful work. However, as is the case with some of the multiple stage Rankin Cycles, if additional heat was added to the system and the steam raised to the superheated state, additional work production or power delivery could be accomplished through the turbine.

At point 5, the vapor and liquid mixture is ready to be condensed fully to saturated liquid phase, represented by point 6.
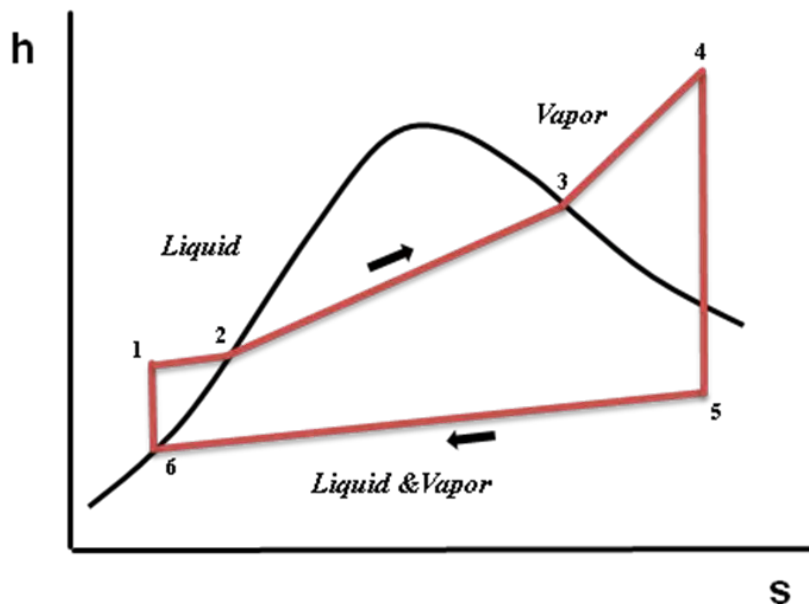
Figure 3.22 Heat Cycle in a Rankin Engine with Superheat, h vs. υ.

Process Flow from Point 5 to 6:

The transition from stage or point 5 to point 6 constitutes the condensation stage of this Rankin Cycle. This transition takes place in the condenser. In this process stage, heat is removed from the vapor and liquid mixture in order to transform the mixture into a saturated liquid.

Transition from point 5 to point 6 is a non-adiabatic process; in that, heat is removed from the system, or the working fluid, by the condenser.

As evident through the pressure vs. specific volume graph in Figure 3.20, the transition from point 5 to point 6 occurs at a constant pressure; in other words, this portion of the Rankin Cycle is *isobaric.* In addition, the transition from point 5 to point 6, results in a substantial reduction of the specific volume. And, since specific volume is the inverse of a density, as the specific volume decreases, the density increases. This observation is supported by the physical transformation of the working fluid from a lower density vapor liquid mixture to a, significantly, denser saturated water.

The temperature vs. entropy graph in Figure 3.21 shows that as this Rankin cycle transitions from point 5 to point 6, the temperature stays constant, or this process is *isothermal*. This comports with the physics associated with general condensation of vapors. Condensation of vapor involves "latent" heat removal and the temperature remains constant as the heat is removed.

Since heat is removed in this stage, there is a change in **Q** and it occurs at a constant temperature. According to Eq. 3.26, restated below, change in heat at a constant temperature results in net change in entropy:

$$\Delta s \ = \ q \ / \ T \text{ abs} \qquad\qquad \textbf{Eq. 3.26}$$

Therefore, in the process transition from point **5** to point **6,** there is a net change in the entropy. This is supported by the **T** vs. **s** graph in Figure 3.21.

Figure 3.22 shows that the transition from point 5 to point 6 also involves some reduction in enthalpy, **h.** This is typically the case when there is a significant change in heat, **Q**.

Process Flow from Point 6 to 1:

The transition from stage or point 6 to point 1 is the last stage in this Rankin Cycle. This stage of the Rankin Cycle boosts the pressure of the working fluid to the level of pressure in the boiler. This increase in the overall pressure head or pressure energy of the fluid is accomplished through the feed pump. The fact that the pressure of the working fluid is enhanced substantially is illustrated through the pressure vs. specific volume graph in Figure 3.20. Note that as the pressure rises significantly in the transition stage between point 6 and point 1, the specific volume υ stays practically constant. Since the Rankin Cycle considered in this analysis recycles the condensate, it could - for most practical purposes - be construed as a ***closed system***. As we learned earlier, in a closed thermodynamic system, the mass remains constant. Then, by definition, as represented mathematically in Eq. 3.27 below, the volume would remain constant in this last stage of the Rankin Cycle.

$$\upsilon \; = \; 1/\rho \qquad\qquad \textbf{Eq. 3.27}$$

Where,

υ = Specific Volume

ρ = Density = Mass /Volume

Note that the working fluid had already arrived at the saturation line – the line on the left half of Figure 3.20 graph that represents saturated liquid – at point 6. So, in the transition from point 6 to point 1, as the pressure is raised by pump, the working fluid, or water, is driven into the ***subcooled*** liquid state. As we noted in the steam table discussion in Segment 1, as the pressure of saturated water is increased, the saturation temperature elevates, as well. Therefore, when the Rankin Cycle is repeated, the first stage, from point 1 to point 6 would require commensurate amount of heat and higher temperature (saturation temperature) to boil the water.

Since no heat is added or removed from the water in this stage, $\Delta Q = 0$, and according to Eq. 3.26, $\Delta s = 0$. In other words, as evident from the **T** vs. **s** and **h** vs. **s** graphs, this stage is ***isentropic***. Note that despite the fact that this stage is isentropic and that no heat is added to the system, transition from point 6 to 1 is ***not*** isoenthalpic (constant enthalpy). The small rise in temperature and

enthalpy in this stage, as exhibited in the temperature vs. entropy and enthalpy vs. entropy graphs, is due to the addition of pump head or pressure energy.

Rankin Cycle Equations:

Some of the mathematical relationships or equations that can be used to analyze and define various parameters in a Rankin cycle are listed below:

$$W_{Turbine} = h_3 - h_4 \qquad \textbf{Eq. 3.28}$$

$$W_{Pump} = \upsilon_6(p_1 - p_6) \qquad \textbf{Eq. 3.29}$$

$$Q_{in} = h_4 - h_1 \qquad \textbf{Eq. 3.30}$$

$$Q_{out} = h_5 - h_6 \qquad \textbf{Eq. 3.31}$$

$$\eta_{Thermal} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{W_{Turbine} - W_{Pump}}{Q_{in}} \qquad \textbf{Eq. 3.32}$$

Where,

$W_{Turbine}$ = Work performed on the turbine by the steam, in BTU's or kJ

W_{Pump} = Work performed by the pump on the vapor, in BTU's or kJ

$\eta_{Thermal}$ = Thermal efficiency of the entire Rankin cycle

Q_{in} = Heat added by the boiler to the working fluid, in BTU's or kJ

Q_{out} = Heat removed from the working fluid by the condenser, in BTU's or kJ

h_1 = Enthalpy at point 1 in BTU/lbm or kJ/kg, from steam tables

h_3 = Enthalpy at point 3 in BTU/lbm or kJ/kg, from steam tables

h_4 = Enthalpy at point 4 in BTU/lbm or kJ/kg, from steam tables

h_5 = Enthalpy at point 5 in BTU/lbm or kJ/kg, from steam tables

h_6 = Enthalpy at point 6 in BTU/lbm or kJ/kg, from steam tables

p_1 = Pressure at point 1, in lbf/ft^2, or Pa, i.e. N/m^2

p_6 = Pressure at point 6, in lbf/ft^2, or Pa, i.e. N/m^2

Case Study 3.1 – Rankin Engine

A thermodynamic system consists of a Rankin engine with superheat function. The enthalpy vs. entropy graph for this system is shown in Figure 3.23 below. The mass flow rate of the system or working fluid is 200 lbm/sec. As an energy engineer, you are to explore the answers to the following questions based on the data provided:

a) If the enthalpy of the fluid is 1500 BTU/lbm and the entropy of the system is approximately 1.8900 BTU/(lbm-°R), what phase or stage is the system in:

(i) Steam generation stage
(ii) Steam superheating stage
(iii) Condensation stage
(iv) Work producing turbine stage

b) If the enthalpy of the superheated steam entering the turbine is 1860 BTU/lbm and the discharged steam has an enthalpy of 1320 BTU/lbm, what is the approximate amount of power delivered to the turbine shaft? Assume that the heat loss is negligible in this system.

(i) 1 MW
(ii) 10 kW
(iii) 50 MW
(iv) 1.3 MW

c) If the enthalpy of the working fluid is 1200 BTU/lbm and the entropy is 1.8100 BTU/(lbm-°R), what is the phase of the working fluid?

(i) Saturated vapor
(ii) Superheated vapor
(iii) A mixture of vapor and liquid
(iv) Sub-cooled liquid

d) If the enthalpy of the working fluid is 1500 BTU/lbm and the entropy is 1.8650 BTU/(lbm-°R), what is the phase of the working fluid?

(i) Saturated vapor
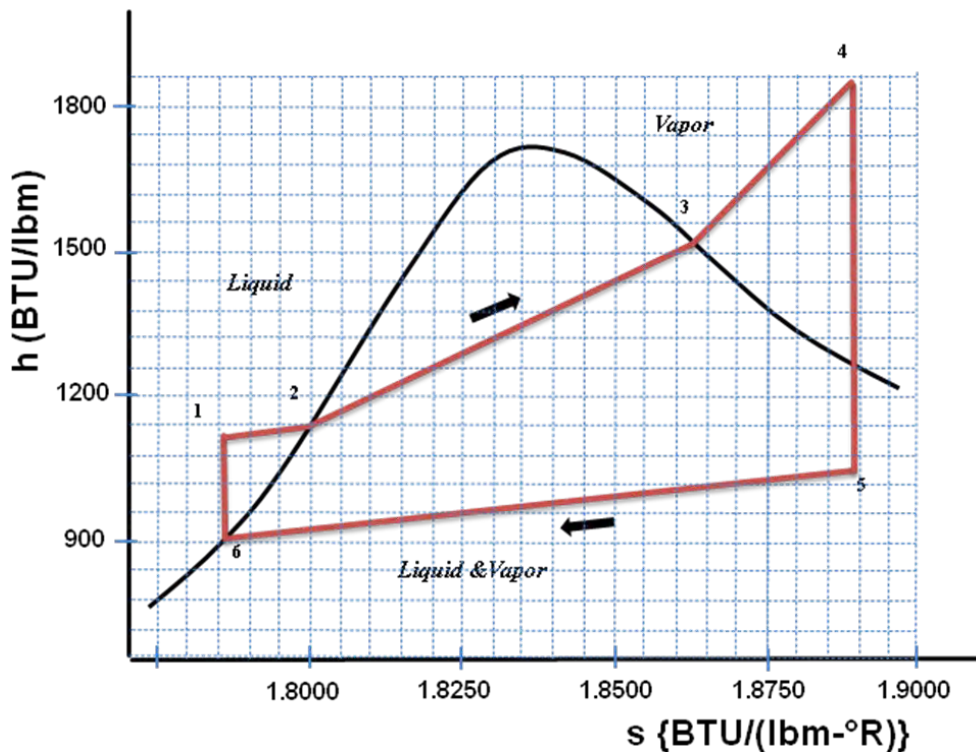(ii) Superheated vapor
(iii) Saturated liquid
(iv) Sub-cooled liquid

Figure 3.23 Heat Cycle in a Rankin Engine with Superheat, h vs. υ. Case Study 3.1

Analyses/Solutions:

a) If the enthalpy of the fluid is 1500 BTU/lbm and the entropy of the system is approximately 1.8900 BTU/(lbm-°R), what phase or stage is the system in?

Answer:

To address this question, we must first locate the point on the graph where **h** = 1500 BTU/lbm and **s** = 1.8900 BTU/(lbm-°R). This point is labeled as point "**a**" on the enthalpy vs. entropy graph in Figure 3.24. Point **a** lies in the region that is clearly outside the saturated vapor curve. In other words, at point **a**, the working fluid is clearly ***superheated steam***. Furthermore, point **a** lies on the transition path between points **4** and **5**. As discussed earlier, the transition path between points **4** and **5** represents the work producing stage in the turbine

Therefore the correct answer would be:

(iv) Work producing turbine stage

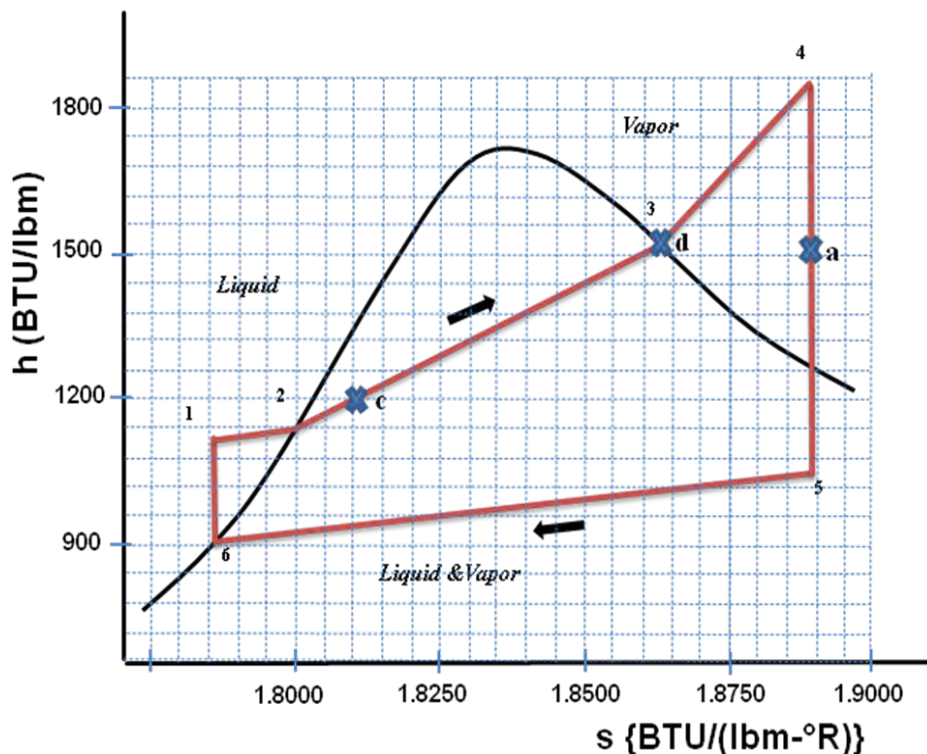**Figure 3.24 Heat Cycle in a Rankin Engine with Superheat, h vs. υ; Case Study 3.1**

b) If the enthalpy of the superheated steam entering the turbine is 1860 BTU/lbm and the discharged steam has an enthalpy of 1320 BTU/lbm, what is approximate amount of power delivered to the turbine shaft if the efficiency of the turbine is 70%? Assume that the heat loss is negligible in this system.

Power delivered by the turbine to the shaft, or generator

$$= (\textbf{Power}_{\textbf{Steam}}) \times (\boldsymbol{\eta}) \qquad \textbf{Eq. 3.33}$$

Where,

$\boldsymbol{\eta}$ = Efficiency of the turbine = **70%** {Given}

Power$_{\textbf{Steam}}$ = Power Delivered by the Steam, to the Turbine Blades

Since we are allowed to assume that there is not heat loss in the turbine, the difference between the enthalpy of the superheated steam entering the turbine and the discharged steam would be equal to the energy {Note: Power = Energy/Time} delivered by the steam to the turbine.

∴ **Power delivered to turbine in BTU's/sec = (h_i – h_f) . ($\dot{m}$)**   **Eq. 3.34**

Where,

$\dot{\textbf{m}}$ = Mass flow rate of the working fluid or water = **200 lbm/sec** {Given}

$\textbf{h}_{\textbf{i}}$ = Enthalpy of steam entering the turbine = **1860 BTU/lbm.**{Given}

$\textbf{h}_{\textbf{f}}$ = Enthalpy of steam discharged = **1320 BTU/lbm** {Given}

Then, application of Eq. 3.34 would yield:

Power delivered to turbine, **in BTU's/sec**
= (1860 BTU/lbm **– 1320** BTU/lbm**) . (200 lbm/sec)**

Or, Power delivered to turbine, **in BTU's/hr**
= (1860 BTU/lbm **– 1320** BTU/lbm**) . (200 lbm/sec) . (3600 sec/hr.)**
= 388,800,000 BTU's/hr

Since there are 3,413 BTU's/hr per kW,

Power delivered to turbine, in kW

= (**388,800,000 BTU's/hr)/ 3413 (BTU's/hr/kW**

= 113,917 kW or 113.92 MW

Then, by applying **Eq. 3.33**:

Power delivered by the turbine to the shaft, or the generator

$$= (\textbf{Power}_{\textbf{Steam}}) \text{ x } (\eta)$$
$$= (\textbf{113.92 MW}) \textbf{ x (0.7)}$$
$$= \textbf{79.74 MW}$$

∴ **The correct answer is (iv) 79.74 MW**

Ancillary to Example 3.1 (b):
As apparent upon examination of Figure 3.23 and Figure 3.24, the change in enthalpy stated in part (b) - from 1860 BTU/lbm to1320 BTU/lbm - occurs while the working fluid *remains in superheated realm*. If the final enthalpy, in this turbine phase of the engine cycle, had dropped below the saturated vapor curve, i.e. point 5, where h = 1020 BTU/lbm, the solution for this part of the problem would have required more steps. This is because at point 5, for instance, the working fluid would be a mixture of liquid and vapor phases and each phase would have a different or distinct enthalpy value. The total final enthalpy in such a case would be a sum of the saturated vapor and liquid enthalpies, added in proportion determined by the humidity ratio, ω.

c) If the enthalpy of the working fluid is 1200 BTU/lbm and the entropy is 1.8100 BTU/(lbm-°R), what is the phase of the working fluid?

Answer:
To address this question, we must locate the point on the graph where **h** = 1200 BTU/lbm and **s** = 1.8100 BTU/(lbm-°R). This point is labeled as point "**c**" on the enthalpy vs. entropy graph in Figure 3.24. Point **c** lies in the region that falls between **saturated liquid line** and the **saturated vapor line**. In other words, at point **c**, the working fluid is in a phase that consists of a mixture of liquid and vapor.

Therefore the correct answer would be:

      (iii) A mixture of vapor and liquid

d) If the enthalpy of the working fluid is 1500 BTU/lbm and the entropy is 1.8650 BTU/(lbm-°R), what is the phase of the working fluid?

Answer:
To address this question, we must locate the point on the graph where **h** = 1500 BTU/lbm and **s** = 1.8650 BTU/(lbm-°R). This point is labeled as point "**d**" on the enthalpy vs. entropy graph in Figure 3.24. Point **d** lies directly on the **saturated vapor line**. In other words, at point **d**, the working fluid is in a phase that consists of entirely of saturated vapor. This occurs as the working fluid begins its transitions from points **3** to **4**; just before it is superheated to point **4**.

Therefore the correct answer would be:

   (i) Saturated vapor

Carnot Cycle:

The Carnot cycle, similar to the Rankin cycle, is a heat engine cycle that converts thermal energy into mechanical and electrical energy. However, unlike the Rankin cycle, the Carnot cycle is and ideal power cycle. In strict context, a Carnot cycle is impractical to implement.

In order to understand the sequential transition of processes or stages within the Carnot cycle, we will examine the **pressure** vs. **specific volume**, **temperature** vs. **entropy** and **enthalpy** vs. **entropy** graphs shown in Figure 3.25, Figure 3.26, and Figure 3.27, respectively. Note that these graphs provide a simplified view of the process mechanics in a Carnot cycle. A less simple but more realistic insight into the Carnot Cycle is presented in Figure 3.28.
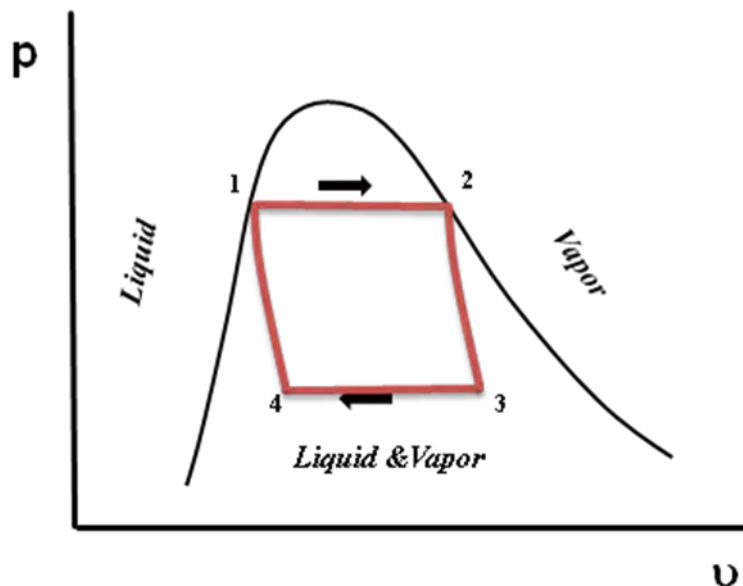
Process Flow from Point 1 to 2:
Point 1 lies on the saturated water line on all three graphs; in Figure 3.25, Figure 3.26, and Figure 3.27. In other words, at point 1, the working fluid is in saturated liquid state on all three graphs. On the other hand, point 2 lies directly on the saturated vapor curve and the working fluid is in the saturated vapor state on all three graphs Therefore, as heat is added to the saturated water at point 1, it transitions to point 2. As evident from Figure 3.25 and

Figure 3.27, this transition from point 1 to point 2 is ***isobaric*** and ***isothermal***. Note that this transition or thermodynamic process from point 1 to point 2 in the Carnot cycle is similar to the transition from point 2 to point 3 in the Rankin cycle. Of course, since ***heat is being added*** to the system or the working fluid, the process flow from point 1 to point 2 is ***non-adiabatic.*** This segment of the Carnot cycle can also be considered as isothermal expansion of the saturated liquid to saturated vapor.

As we examine the **T** vs. **s** graph, in Figure 3.26, we note that, while the process flow from point 1 to point 2 is isothermal or constant temperature, there is a distinct increase in the entropy (**s**).

The transition from point 1 to point 2, from the enthalpy (**h**) and entropy (**s**) perspective, can be observed in Figure 3.27, to entail the pronounced rise in the entropy accompanied by a notable increase in the enthalpy.


Figure 3.25 Heat Cycle in a Carnot Engine, **p** vs. ʋ.

Process Flow from Point 2 to 3:

The transition from stage or point 2 to point 3 would, theoretically, take place in the work producing segment of the Carnot cycle. Note that, as apparent from Figures 3.25, 3.26 and 3.27, unlike the corresponding point in the Rankin cycle, point 2 in the Carnot cycle is ***not in the superheated steam*** realm. As the saturated steam transfers its energy to the turbine, its temperature, enthalpy and pressure drop to levels represented by point 3 on the pressure (**p**), specific volume (υ), temperature (**T**), enthalpy, (**h**) and entropy (**s**) diagrams.

As exhibited in the enthalpy vs. entropy (Figure 3.27) and the temperature vs. entropy graphs (Figure 3.26), the entropy remains constant in this stage, thereby making this stage isentropic. Note that the heat is neither added nor removed from the vapor – the working fluid – in this stage. Therefore, this stage is not just ***isentropic*** but also ***adiabatic.*** This segment of the Carnot cycle can also be considered as ***isentropic expansion*** of vapor. This is because the entropy stays constant and the pressure drops substantially as the working fluid transitions from point 2 to point 3. The thermodynamic process diagram in Figure 3.26 shows that, as the isentropic expansion of the vapor in this stage of the Carnot cycle performs work on the surroundings and transfers energy in form of work, it results in lower vapor temperature, **T_Low**.
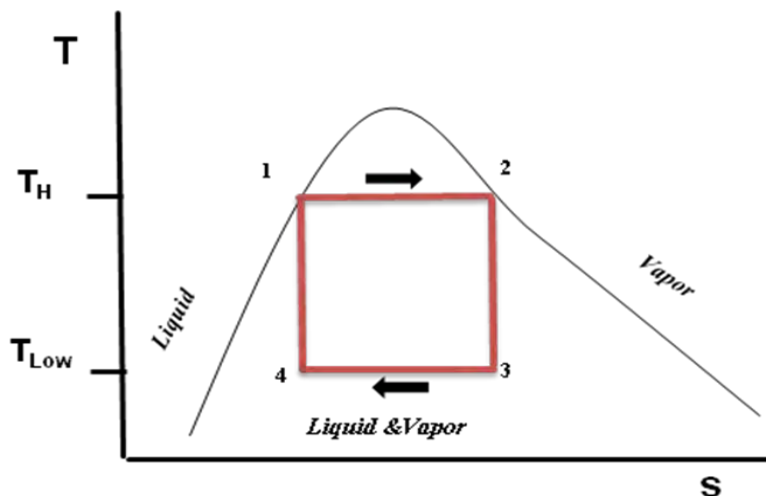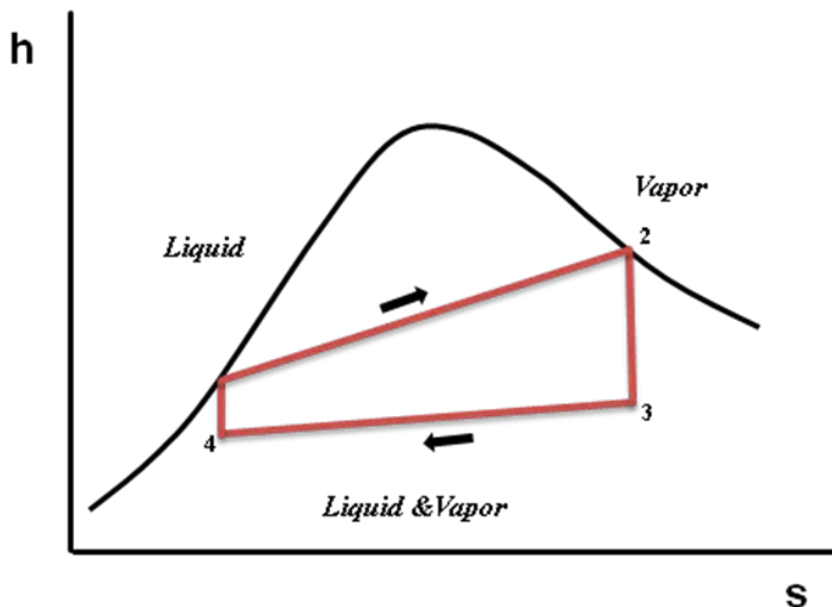

Figure 3.26 Carnot Heat Cycle, **T** vs. **s.**

Process Flow from Point 3 to 4:

The transition from point 3 to point 4 consists of isothermal compression of the lower temperature vapor. The fact that this stage of the Carnot cycle is isothermal is obvious from Figure 3.26. However, the fact that vapor is compressed in this stage is not clearly supported by the simplified pressure vs. specific volume graph shown in Figure 3.25. The graph is Figure 3.25 implies that the pressure is constant in this stage. Examination of the more realistic pressure vs. specific volume graph, in Figure 3.28, shows that the pressure of the working fluid does rise in this stage of the Carnot cycle.

In the transition from point 3 to point 4, some heat is removed from the working fluid or the system, thus rendering this segment of the Carnot cycle non-adiabatic. In other words, Q is negative in this stage. Heat is removed from the system in order to maintain the vapor at a constant temperature as work is performed by the surroundings, namely the compressor pump, onto the system. The removal of heat in this stage comports with the small drop in the enthalpy, as shown in Figure 3.27.

Figure 3.27 Carnot Heat Cycle, **h** vs. **s.**

Since heat is removed in this stage, there is a change in **Q** and it occurs at a constant temperature. According to Eq. 3.26, restated below, change in heat at a constant temperature results in net change in entropy:

$$\Delta s \;=\; q \,/\, T_{abs} \hspace{3cm} \text{Eq. 3.26}$$

Therefore, in the process transition from point **3** to point **4,** there is a net change in the entropy. This is supported by the **T** vs. **s** graph in Figure 3.26.
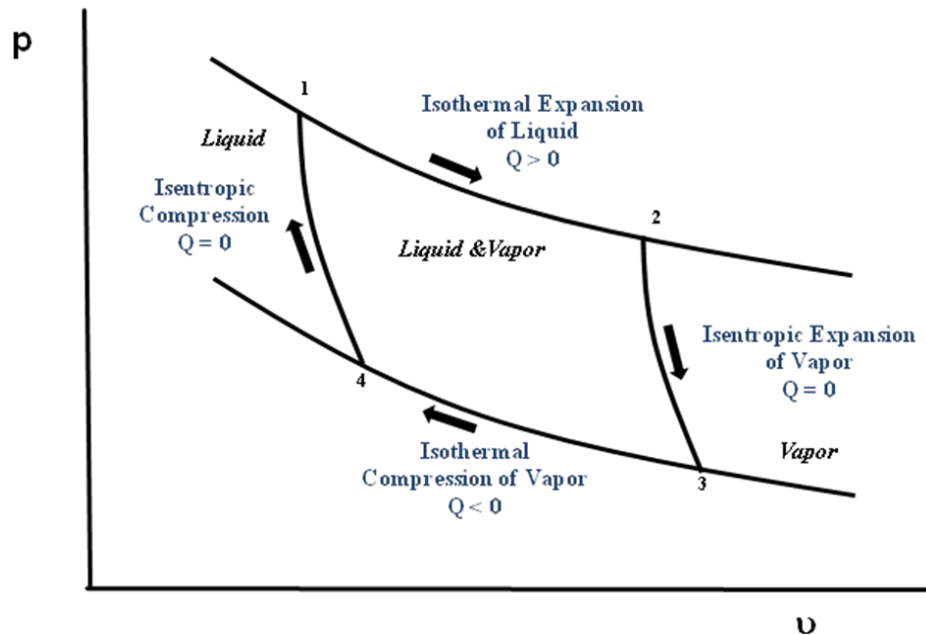

Figure 3.28 Alternative Representation of Carnot Heat Cycle, **p** vs. ʋ**.**

Process Flow from Point 4 to 1:

The transition from stage or point 4 to point 1 is the last stage in this Carnot Cycle. This stage represents an isentropic compression process. As apparent from Figures 3.33 and 3.34, the transition from point 4 to point 1 occurs at a constant entropy **s**. This stage of the Carnot cycle boosts the pressure of the working fluid to the level of pressure in the boiler. The fact that the pressure of the working fluid is enhanced substantially is illustrated through the pressure vs. specific volume graphs in Figures 3.25 and 3.28. In an ideal system, it is assumed that no heat is either added or removed from the system in the transition path from 4 to 1. Therefore, this process from point 4 to point 1 is an adiabatic process. In other words, Q = 0 in the last stage of the Carnot cycle.

Carnot Cycle Equations:

Some of the mathematical relationships or equations that can be used to analyze and define various parameters in a Carnot cycle are listed below:

$$W_{Turbine} = h_2 - h_3 \qquad \text{Eq. 3.35}$$

$$W_{pump} = h_1 - h_4 \qquad \text{Eq. 3.36}$$

$$Q_{in} = h_2 - h_1 \qquad \text{Eq. 3.37}$$

$$Q_{out} = h_3 - h_4 \qquad \text{Eq. 3.38}$$

$$\eta_{Thermal} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{W_{s-Turbine} - W_{s-Pump}}{Q_{in}} \qquad \text{Eq. 3.39}$$

Where,

$W_{Turbine}$ = Work performed on the turbine by the steam, in BTU's or kJ

W_{Pump} = Work performed by the pump on the vapor, in BTU's or kJ

$W_{s-Turbine}$ = Work performed on turbine by the steam, in BTU's or kJ, during the isentropic process from point 2 to 3.

W_{s-Pump} = Work performed by the pump on the vapor, in BTU's or kJ, during the isentropic process from point 4 to 1.

$\eta_{Thermal}$ = Thermal efficiency of the entire Rankin cycle

Q_{in} = Heat added by the boiler to the working fluid, in BTU's or kJ

Q_{out} = Heat removed from the working fluid by the condenser, in BTU's or kJ

h_1 = Enthalpy at point 1 in BTU/lbm or kJ/kg, from steam tables

h_2 = Enthalpy at point 2 in BTU/lbm or kJ/kg, from steam tables

h_3 = Enthalpy at point 3 in BTU/lbm or kJ/kg, from steam tables

h_4 = Enthalpy at point 4 in BTU/lbm or kJ/kg, from steam tables

h_5 = Enthalpy at point 5 in BTU/lbm or kJ/kg, from steam tables

h_6 = Enthalpy at point 6 in BTU/lbm or kJ/kg, from steam tables

p_1 = Pressure at point 1, in lbf/ft², or Pa, i.e. N/m²

p_6 = Pressure at point 6, in lbf/ft², or Pa, i.e. N/m²

Comparison between Rankin and Carnot Cycles:

A basic Rankin Cycle is similar to the Carnot Cycle with the following exceptions:

1) In the Rankin cycle, the compression process occurs in the liquid region. The compression process in a Carnot cycle takes place in the region where working fluid exists in vapor and liquid mixture form. This is apparent from examination of the two, comparative, temperature vs. entropy graphs shown in Figure 3.29. Compression of the working fluid in liquid form is more efficient than the compression and pumping of the vapor and liquid mixture.
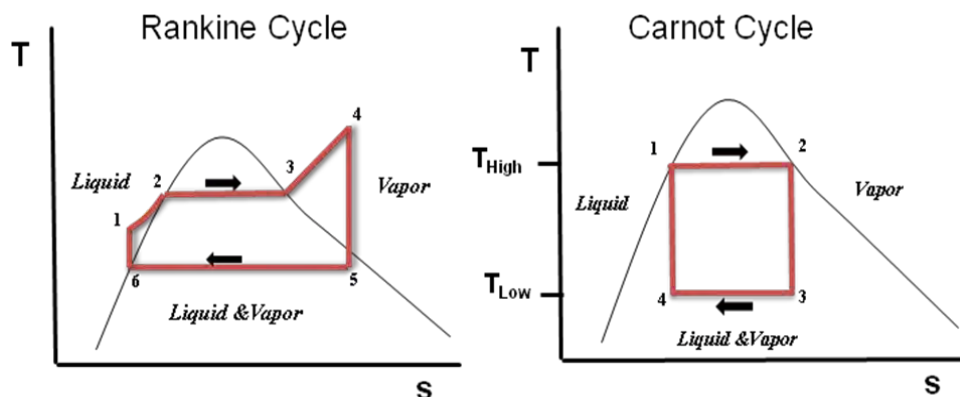
Figure 3.29 Rankine and Carnot Cycle Comparison, Temperature vs. Entropy

2) Rankin cycle is closely approximated and applied in steam turbine plants.

3) The working fluid crosses through the saturated liquid line and is condensed completely into the subcooled phase in the Rankin cycle. In the

Carnot cycle the working fluid is compressed only *up to* the saturated liquid line.

4) On the high enthalpy spectrum of the process, the working fluid is heated well into the superheated phase area in the Rankin cycle; whereas in the Carnot cycle, the working fluid only heats *up to* the saturated vapor line.

Other Major Types of Cycles:

Aside from the simple Rankin cycle, Rankin cycle with superheating, and the Carnot cycle discussed above, some other cycles that find common applications are as follows:

1) Rankin Cycle with Superheat and Reheat
This type of heat engine or cycle is similar to the Rankin cycle with a superheating function, except that it includes a reheat feature as well. See Figure 3.30. The reheat feature allows for the steam to be reheated after its first pass through the turbine. This creates a second enthalpy differential and an additional opportunity to produce work or deliver energy to the turbine blades.

Rankine Cycle with Superheat and Reheat
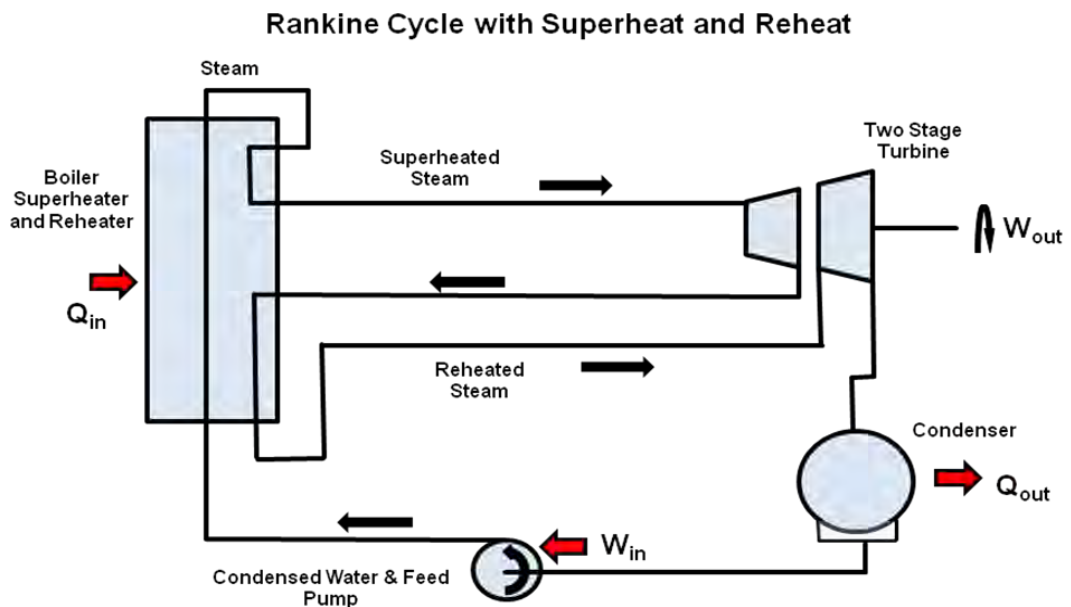
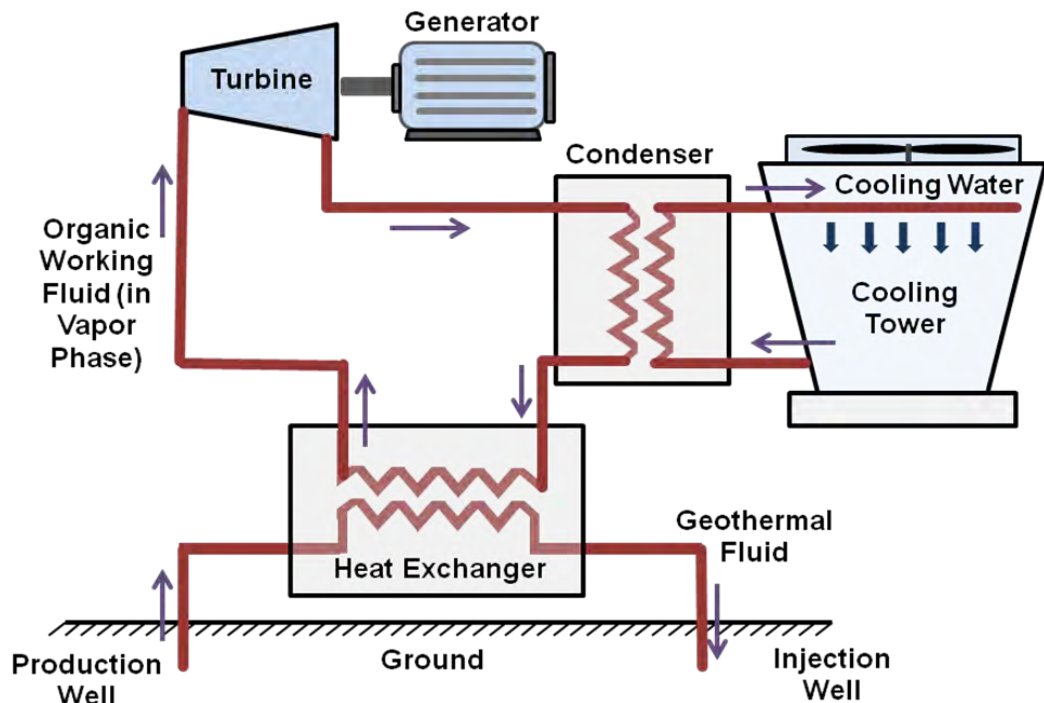**Figure 3.30 Working Principle of a Rankin Cycle Engine with Superheat and Reheat**

2) Rankin Cycle with Regeneration

This type of Rankin cycle allows the condensate to be preheated using bleed stream from multiple points in the turbine. This preheated condensate water is then pumped to the boiler. Rankin cycles with regeneration may or may not be equipped with a superheat stage. Regenerative Rankin cycles are typically equipped with multiple feedwater heaters.

3) Binary Cycle

A binary cycle engine is, in essence, a Rankin cycle engine that utilizes two different fluids. For example, in geothermal type binary cycle engines, as the one shown in Figure 3.31, these two fluids could be geothermal fluids, such as water and some type of organic working fluid, like pentane hydrocarbon or butane.

Figure 3.31 Working Principle of a Binary Cycle System

4) Cogeneration and Combined Cycles

Most heat engines operate at efficiencies that are less than 50%. So, if the heat engine efficiency is 35%, it would mean that 65% of the heat is ***not*** converted to work; instead it is discarded or dissipated into the ambient atmosphere as waste heat. The heat lost in the heat engines is referred to as ***rejected energy*** or ***rejected heat***. Cogeneration and combined cycle systems harness the rejected heat for various purposes, thus enhancing the efficiency of the overall system.

Cogeneration: When waste heat is recovered and used to produce hot water or for space heating purposes, also referred to as ***district heating***, the process is termed as cogeneration. In cogeneration systems the recovered heat is not directly used to generate electricity. If the heat recovered through a cogeneration system is in the form of steam, steam that is a byproduct of an electric power generation system, then it is called a ***CHP***, or ***Combined Heat and Power Cycle***. The temperature of steam applied in cogeneration systems ranges from 80°C to 180°C. The cogeneration steam is not only used for space heating but can also be utilized for cooling purposes through the absorption process or absorption chillers.

Combined Cycle: If recovered waste heat from a power generating system is used to vaporize water and produce steam, and if that steam is subsequently used to generate electric power (once again), the process is referred to as a combined cycle.

Working principle behind a typical combined cycle is illustrated in Figure 3.32. As apparent from this diagram, generator No. 2 is powered by the steam produced through heat transfer in the primary generator's effluent stack.

Working Principle of a Combined Cycle System
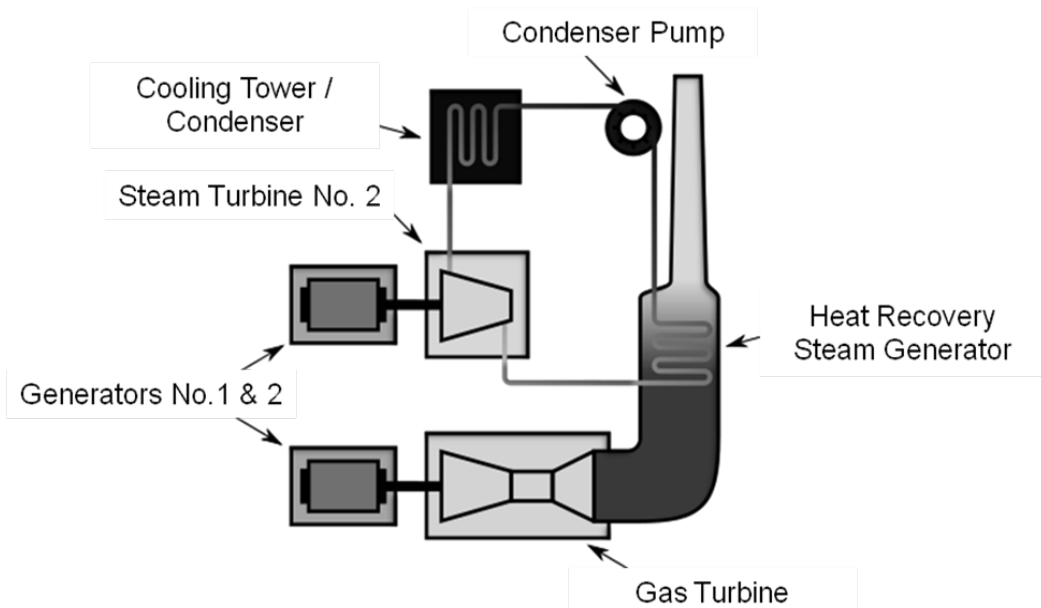
Figure 3.32 Working Principle of a Combined Cycle System

Self-assessment Problems and Questions – Segment 3

1. Ideal heat engine always include a boiler with superheating function.

A. True
B. False

2. In the heat engine represented by the enthalpy vs. entropy graph in Figure 3.22, the heat is added to working fluid in:

(i) Steam generation stage
(ii) Steam superheating stage

(iii) Condensation stage

(iv) Steam generation stage and the Steam superheating stage

3. In the heat engine represented by the enthalpy vs. entropy graph in Figure 3.22, the energy contained in the superheated working fluid is converted into the rotational kinetic energy in:

(i) Process transition from point 2 to 3

(ii) Process transition from point 4 to 5

(iii) Process transition from point 6 to 1

(iv) Process transition from point 1 to 2

4. A thermodynamic system consists of a Rankin engine with superheat function. The enthalpy vs. entropy graph for this system is shown in Figure 3.33 below.
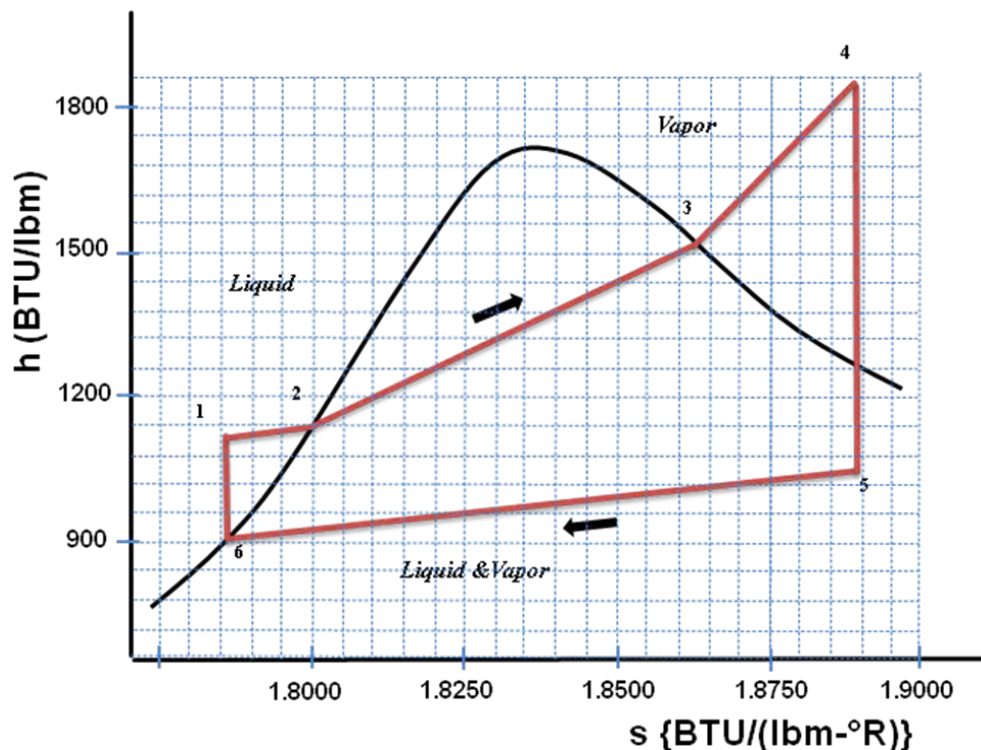

Figure 3.33 Heat Cycle in a Rankin Engine with Superheat, h vs. s.

The mass flow rate of the system or working fluid is 100 lbm/sec. Answer the following questions based on the data provided:

a) If the enthalpy of the fluid is approximately 1850 BTU/lbm and the entropy of the system is approximately 1.8900 BTU/(lbm-°R), what phase or stage is the system in?

      (i) Work producing turbine stage
      (ii) Steam superheating stage
      (iii) Condensation stage
      (iv) Steam generation stage

b) If the enthalpy of the subcooled water entering the boiler is 900 BTU/lbm and the enthalpy of the water at a downstream point in the boiler is 1080 BTU/lbm, what is approximate amount of heat added in **MMBTU per hour**? Assume that there is no heat loss.

      (i) 1 MMBTU/hr
      (ii) 1.08 MMBTU/hr
      (iii) 50 BTU/hr
      (iv) 1.3 MMBTU/hr

c) If the enthalpy of the working fluid is 1440 BTU/lbm and the entropy is 1.8500 BTU/(lbm-°R), what is the phase of the working fluid?

      (i) Saturated vapor
      (ii) Superheated vapor
      (iii) A mixture of vapor and liquid
      (iv) Sub-cooled liquid